NEW SERIES, NO. 151 (VOL. XX) SEPTEMBER, 1925

JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION

Formerly the Quarterly Publication of the American Statistical Association

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Published quarterly by the American Statistical Association

Rumford Press Building, Concord, N. H.

Editorial Office, New York City

Entered at the post-office, Concord, N. H., as second-class mail matter

Acceptance for mailing at the special rate of postage provided for in Section No. 1103, Act of October 3, 1917, authorized April 29, 1922

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NEW SERIES, NO. 150 (VOL. XX) SEPTEMBER, 1925

JOURNAL OF THE AMERICAN STATISTICAL ASSOCIATION

Formerly the Quarterly Publication of the American Statistical Association

ON THE TRUE RATE OF NATURAL INCREASE[1]

AS EXEMPLIFIED BY THE POPULATION OF THE UNITED STATES, 1920

By Louis I. Dublin and Alfred J. Lotka, *Metropolitan Life Insurance Company, New York*

INTRODUCTION

The present policy of restricting immigration into the United States lends a particular interest to inquiries into the powers of natural increase of our population. To the casual observer it may appear that the excess of our birthrate over our deathrate gives us a measure or at least an index of this power of natural increase. If he finds that in 1920 there was an annual birthrate of 23.40 per thousand and a death-rate of 12.41 per thousand, he may feel that there is no occasion for solicitude in the face of a balance of 10.99 representing the "natural rate of increase" of the population.[2]

As a matter of fact this argument is misleading. Both birthrate and deathrate are affected by the age distribution of the population. If there is present a relatively high proportion of persons between the ages of 20 and 45, this will tend to give a high birthrate because at these ages reproduction takes place; and it will tend to give a low deathrate because at these ages mortality is at a low point.

[1] *Note:* For the convenience of those readers who do not care to follow in detail the mathematical treatment of the problems here contemplated, the exact and rigorous methods by which the results were obtained have been separated from the main text and described in the *Appendix*. This section has been prepared in its entirety by Alfred J. Lotka. In the text, simple methods for which mathematical rigor is not claimed, but which give approximate values, have been indicated. In each case the exact value is also given for comparison, so that the reader can currently control the degree of departure from accurate values. While the approximate methods are useful for the purpose of simple exposition, they would, of course, be of little value without the endorsement and control of the results of the rigorous analysis. Meanwhile, the reader averse to mathematics will here find in available form all the results, and much of the argument, collected together in the first part of the paper, which to this extent forms a complete and independent whole.

[2] For reasons that will appear in what follows (see p. 307) the figures here selected for quotation are those relating to the births and deaths of white females, expressed in reation to the white female population of the Birth Registration Area. Figures so stated are, of course, closely similar to birthrates and deathrates as commonly stated.

It is true that for certain economic reasons this would be a desirable state of affairs, provided that the high proportion of persons in mid-life were guaranteed to continue. Persons at these ages do productive work, and the presence of a high proportion of them acts favorably on the general level of well-being of the whole community. But an examination of the reasons for this preponderance of persons in the productive period of life does not seem to give much support for optimism as to the continued prevalence of this state of affairs. For there are essentially two circumstances that have given us this relatively high proportion of persons in and about mid-life. First, immigration, which brings to us chiefly adults, and thus swells at once the numbers of the adult class of our population. This source of productive and reproductive members of our commonwealth is to be henceforth restricted, so that in time we must expect, on this count, a less marked preponderance of this age class in the population.

The second source of our high adult population is not quite so obvious. Through past decades the birthrate has been falling. This means that persons 30 years old today are survivors out of a relatively[1] larger batch of births, than persons 10 years old, for example, who were born at a time of more recent, and therefore lower birthrate. In this way the older age groups are today weighted with the fruits of a relatively higher birthrate in the more remote past. So long as the birthrate continues falling, this situation of a swollen age distribution at adult ages may not give any outward signs of its inherent weakness. But the birthrate cannot continue to fall indefinitely. Sooner or later it must become stationary or nearly so. The second source of our relatively high proportion of persons in the reproductive age groups will then also be cut off, and then, with precisely the same mortality as today, and precisely the same fecundity of our women, we should have a birthrate very different from that now observed. That is to say, if the present schedule of frequency of maternity for women at each age remained permanently established; and if similarly the present schedule of mortality at different ages continued permanent, then, after the age distribution had had time to adjust itself to these *constant* biological conditions,[2] we should have, instead of the present birthrate of 23.4 per thousand per annum, a birthrate of only 20.9; at the same time the

[1] *I. e.* in proportion to the total population at the time of their birth.

[2] Perhaps it may be well to draw attention to the fact that the difference between the existing state of affairs and that here serving as a basis of comparison, lies merely in the adventitious factor of age distribution. The essential, fundamentally biological factors are strictly the same in the actual and in the standard of comparison. And, whereas the existing age distribution represents a transitory state of affairs incapable of continued existence under constant conditions of maternity frequency and mortality, the age distribution of the standard of comparison is that which is stable, self-sustaining, under the biological conditions *prevailing now*.

deathrate would rise from the present (1920) figure 12.41 to 15.43. Each of these changes alone is significant enough; but when we consider the effect upon the natural rate of increase, that is, upon the excess of the birthrate over the deathrate, we find that this has changed from 11 per thousand per annum to 5.5 per thousand per annum; it has been cut down to just one half!

Since the method of computing the age distribution, birthrate, deathrate and natural rate of increase of a population having a given age schedule of fecundity and of mortality is not very generally known, it will be convenient to illustrate it by a numerical example.

ARGUMENT AND METHOD

In the argument and the computations that follow it will be convenient to restrict our considerations to females. For we shall accept the present figure of 1.06 as the ratio of male to female (white) births and suppose that this remains constant. The figures actually observed [1] for this ratio are as follows:

1917–22	*1922*	*1921*	*1920*	*1919*	*1918*	*1917*
1.060	1.058	1.061	1.059	1.059	1.060	1.060

It is seen that the ratio is essentially constant. In consequence of this a simple relation exists between the figures here computed directly for females, and the corresponding figures for males. This point will be duly referred to when occasion arises, but in the meantime, it relieves us of the necessity of keeping track of both sexes and thus greatly simplifies our reflections.

The general trend of the argument to be followed, and the fundamental data on which computations will be based, are as follows:

Data. (1) The age schedule of fecundity, *i. e.* the maternity frequency $m(a)$ at several ages a.

(2) The age schedule of mortality, *i. e.* a life table, giving survivals $p(a)$ to several ages a.

From these will be deduced

(a) The ratio R_0 of total births in two successive generations.

(b) The mean length T of one generation.

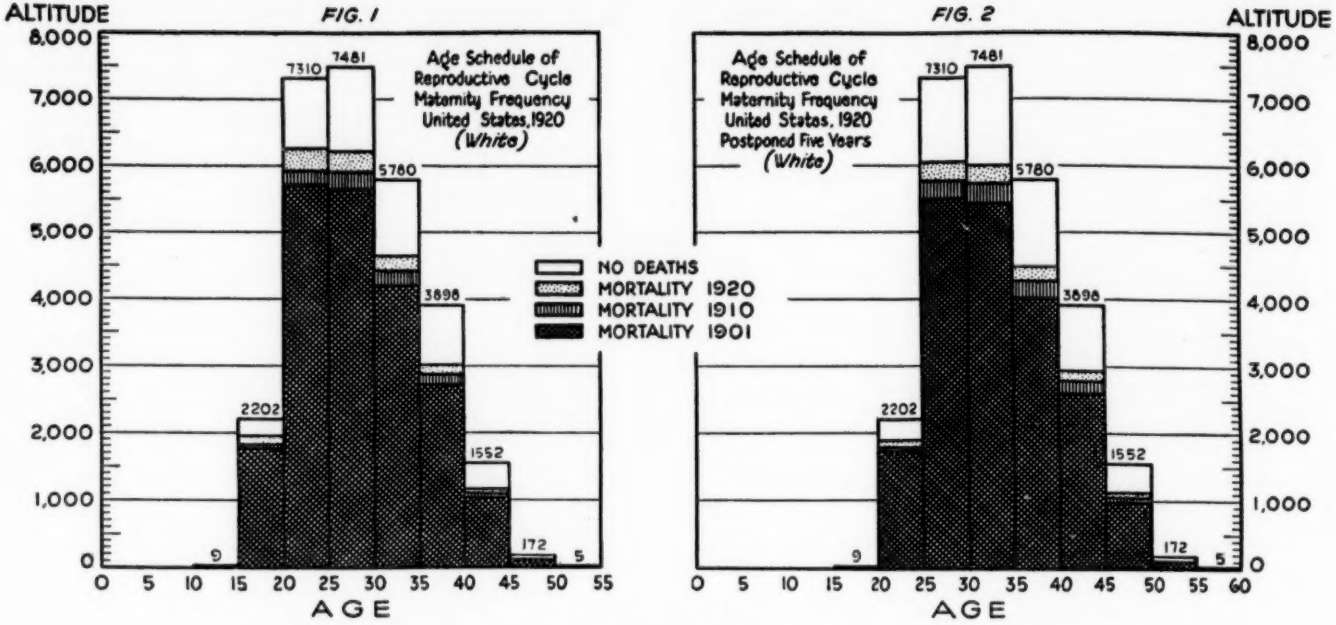
(c) The annual rate of increase r.

(d) The corresponding birthrate and deathrate.

These several items will now be taken up in turn.

Age Schedule of Fecundity. The age schedule of fecundity for females in the United States in 1920 is exhibited in Figure 1, the

[1] U. S. Census Bureau, *Birth Statistics*, 1922, p. 30.



FIGURES 1 AND 2

TOTAL BIRTHS OF DAUGHTERS FROM A COHORT OF 100,000 FEMALES STARTING LIFE TOGETHER

Areas of complete columns (five times altitude) represent potential births of daughters within the age limits indicated, if there were no deaths in the cohort. Areas of truncated columns represent actual births of daughters within same age limits, as reduced by deaths of potential mothers according to life tables of 1920, 1910 and 1901, respectively. Figures indicate altitudes, not areas. To obtain maternity frequencies multiply births of daughters by 2.06 so as to include sons.

uppermost stepped curve (histogram), and Table I, column 2, on the basis of data found in the Birth Statistics for 1920 published by the Census Bureau, pp. 168, *et seq.*

TABLE I

AGE SCHEDULE OF FECUNDITY U. S. WHITE FEMALES, 1920

Maternity frequency $m(a)$ = daughters per annum, per 100,000 white females of all marital states and in stated age groups

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Age groups	$m(a)$	$p(a)$* at pivotal ages 12.5, 17.5, etc.			Product $p(a)$ $m(a)$		
	1920	1920	1910	1901	1920	1910	1901
10–14	9	.88567	.83556	.81232	8	8	7
15–19	2,202	.87438	.82495	.79926	1,950	1,817	1,760
20–24	7,310	.85509	.80852	.77834	6,251	5,910	5,690
25–29	7,481	.82960	.78801	.75268	6,206	5,894	5,630
30–34	5,780	.80181	.76480	.72445	4,634	4,421	4,187
35–39	3,898	.77417	.73839	.69474	3,018	2,878	2,708
40–44	1,552	.74664	.70903	.66336	1,159	1,101	1,030
45–49	172	.71610	.67567	.62917	123	116	108
50–54	5	.62937	.63490	.58894	3	3	3
	28,409 ×5				23,352 ×5	22,148 ×5	21,123 ×5
Total daughters per life time	142,045				116,760	110,740	105,615
With deaths as of	0†				1920	1910	1901

* $p(a)$ is the probability, at birth, of reaching age a. It is the function tabulated in life tables as l_x, if construed as referred to a radix of unity.
† No deaths to end of reproductive age group.

From this it will be seen that according to the present age schedule of fecundity, 100,000 females, if they all survived to the age of 55, would have an aggregate of 142,045 daughters.

Ratio of Total Births in Two Successive Generations. In actual fact, of course, a certain proportion of these females die before completing the reproductive period, some even before entering it. So, for example, between the ages 25 and 30, the 100,000 females originally born would, if they all survived, have 7,481 daughters. But in point of fact only 82,960 out of 100,000 born reach the age of 27.5 years. Consequently, the 100,000 females under consideration will give birth, between the ages of 25 and 29, only to $7{,}481 \times 0.8296 = 6{,}206$ daughters.

Similar remarks apply to the other age groups. The corresponding figures for successive age groups are given in Table I, column 6, and Figure 1, the second[1] stepped curve (histogram) counting from above.

[1] The third and fourth of the stepped curves, and the figures affixed to them, give the corresponding number of daughters if maternity frequency of 1920 had been combined with mortality of 1910 and 1901 respectively. For further discussion of this point see p. 325.

The net result is that if we follow the history of 100,000 females at the current rate of fecundity we find that throughout their life they give birth to 116,760 daughters; or, on an average, one female gives birth to 1.168 daughters in the course of her life. This, then, is the ratio of the total births (of daughters) in two successive generations. It will be convenient for future reference to denote this ratio by the symbol R_o.

Mean Length of One Generation. Since the births extend from about the fifteenth to about the fifty-fifth year of life, it is not possible to designate any one age as *the* age of reproduction, and thus to fix in this simple manner the length of a generation. The term "mean length of one generation" is, in fact, ordinarily used loosely with no attempt at a strict definition. We may define the mean length of one generation as

FIG. 3

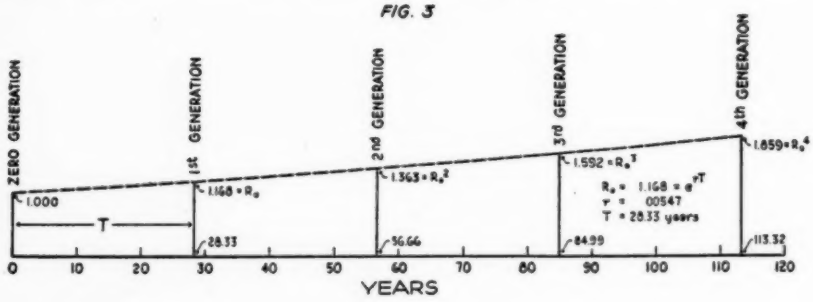


FIGURE 3

DIAGRAM ILLUSTRATING DEFINITION OF MEAN LENGTH OF ONE GENERATION

Secular rate of increase being 0.547 per cent or .00547 per head per annum, and total numbers in successive generations being in ratio of 1.168, these generations must be centered 28.33 years apart to give the stated secular rate of increase. This space of 28.33 years is the mean length of one generation (from mother to daughter).

follows: Starting with a cohort of N individuals (the "zero" generation) all born at the same time, let $R_o N$, $R_o^2 N$, $R_o^3 N$, etc., be the total number of births in the 1st, 2nd, 3rd, etc., generation of their descendants. Actually the births in each generation are spread over a number of years. We may, however, consider them concentrated, for each generation, at one point of time, successive generations being spaced evenly T years apart. There is one particular value of T, one particular spacing of successive generations, which will give the same resultant secular rate of increase as that which actually occurs with the births in each generation spread over a range of years. This particular value of T is the mean length of one generation.

Perhaps the diagram of Figure 3 may help further to elucidate the nature of the quantity thus defined.

The exact method of computing the mean length of a generation is given in the Appendix. Here it will suffice to state that a fair approximation is obtained very simply by finding the median age [1] in the second curve (counting from above) in Figure 1. This median is found to lie at age 27.8 and this, then, gives approximately the mean length of one generation.

A very close approximation to the true value is given by the foot of the perpendicular through the center of gravity of the area under the curve. This comes at 28.46 years. The mean length of one generation thus receives a mechanical interpretation which is brought out graphically in Figures 4 and 5.

The exact method of computation given in the Appendix leads to the value 28.33.

It is to be particularly noted that the mean length of one generation here discussed is that from mother to daughter, and is not the same as that from father to son, owing chiefly to the later marriage of men as compared with women. A separate computation gave for the mean length of one generation from father to son 32.76 years, four and one-half years in excess of the generation from mother to daughter.

Standardized Rate of Increase. It can be shown [2] that with a fixed schedule of maternity rates for women [3] of different ages, and a fixed age schedule of mortality, the population will (in the absence of immigration) gradually settle down to a fixed age distribution. The natural rate of increase r will then be constant. The rate of increase thus defined we will speak of as the *"standardized" or "stable natural rate of increase"* under the given conditions of maternity and mortality. An exact determination of its value requires the solution of a transcendental equation. The method will be found in the Appendix. Here it will suffice to note that a good approximation is very simply obtained from the approximate value of the mean length T of one generation, determined above, and the ratio R_o of total births in two successive generations. For a rate of increase R_o per T years at compound interest corresponds to an annual increase r compounded for T years, thus

$$(1+r)^T = R_o$$
$$r = \sqrt[T]{R_o} - 1.$$

[1] This median age is such that one-half the births in the course of a woman's life time take place below, and one-half above that age, on an average.

[2] *Phil. Mag.*, April, 1911, p. 435; *Proc. Natl. Acad. Sci.*, 1922, Vol. 8, p. 339. A. J. Lotka, *Elements of Physical Biology*, Williams & Wilkins, 1925, p. 112.

[3] The paternity rates for men at different ages do not represent an independent factor, but must be regarded as determined by established habits in assortative marriage according to age, once the remaining factors here considered are fixed.

FIG. 4

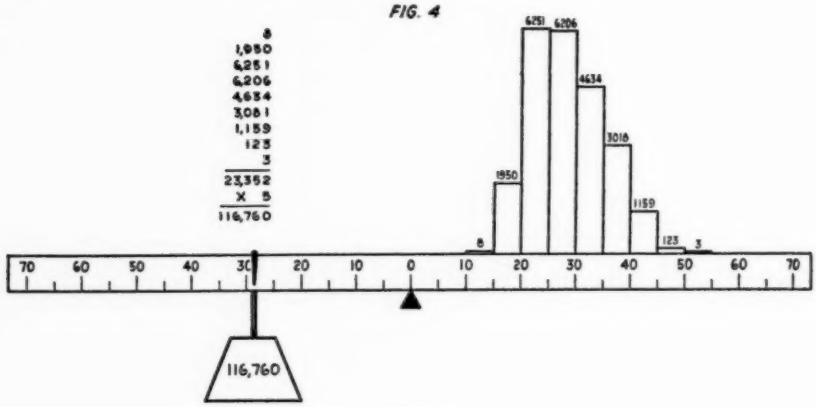


FIGURE 4

MECHANICAL INTERPRETATION OF MEAN LENGTH OF ONE GENERATION

Histogram of daughters from cohort of 100,000 mothers starting life together is balanced by sum of total daughters, 116,760 at 28.46 years from fulcrum. Exact value of mean length of one generation is 28.33, the difference being due to the influence of second and higher moments.

FIG. 5

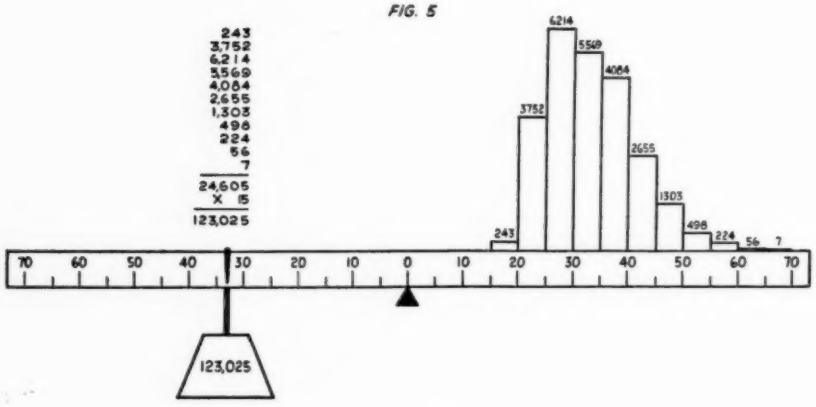


FIGURE 5

MECHANICAL INTERPRETATION OF MEAN LENGTH OF ONE GENERATION

Histogram of sons from cohort of 100,000 fathers starting life together is balanced by sum of total sons, 123,025 at 32.98 years from fulcrum. Exact value of mean length of one generation is 32.76 years.

With the values here under consideration, namely, an increase of 1.168 in the course of a generation of 28.46 years, this gives

$$r = \sqrt[28.46]{1.168} - 1.$$
$$= .00546$$

The exact value found by the method given in the Appendix is

$$r = .00547.$$

It may here be briefly noted that, whereas in the present approximate computation we have first found an approximate value of T and from this have deduced r; in the exact computation this process is reversed, an exact value for r being first obtained, and from this is deduced the exact value of T, in complete accord with the definition of this quantity, as given above.

Standardized Birthrate and Deathrate. When the stable age distribution is established, birthrate and deathrate assume determinate values. The method for their exact computation is given in the Appendix.

A good approximation to the "standardized" or "stable" birthrate (under given age schedules of maternity frequency and mortality) can be obtained by the following comparatively simple argument.

If r, the rate of increase, is small, then the "stable" age distribution will not differ greatly from that of a stationary population, that is, of a "life table population."

Now in a life table population the birthrate per head figured on the present-living (female) population is the same as the birthrate per head figured on a cohort of females starting out in life together and traced through their life. But this latter figure we have already determined, and found to be 1.168 per mean length of life (57.52 years), for we have found that according to 1920 conditions 1,000 females (counted at birth) will in the course of life have 1,168 daughters. Hence the birthrate per annum is $\frac{1.168}{57.52} = .0204$, or 20.4 per thousand. The exact method of computation set forth in the Appendix, which is independent of any simplifying assumptions, gives the value 20.90.

The deathrate now follows immediately from the relation $b - d = r$, which gives, by the approximate method, $d = 14.9$ per thousand; and by the exact method, $d = 15.4$ per thousand.

COMPARISON OF "STANDARDIZED" WITH OBSERVED DEMOGRAPHIC FEATURES

Collecting our results obtained so far, and comparing them with the corresponding observed figures we have

TABLE II
"STANDARDIZED" OR "STABLE" BIRTHRATES AND DEATHRATES COMPARED WITH ACTUAL

	Calculated	Actual in 1920
Birthrate b	20.90	23.40
Deathrate d	15.43	12.41
Natural rate of increase $r=b-d$	5.47	10.99

The figures thus computed differ so markedly from the observed values, that the mind immediately demands a comparative inquiry into the sources of the divergence.

Such sources are, primarily, two in number, namely, 1. Immigration; 2. Declining birthrate.

1. *Immediate Effect of Immigration.* We may approach the inquiry into the effect of immigration upon the birthrate and the natural rate of increase of the population from two different avenues.

(a) We may compare the actual, effective age distribution of the total population from which births actually spring, with that of the native-born population alone, upon whom we should have had to depend for parents of the rising generation if there had been no immigration in recent years. What interests us here are the age groups within the reproductive period. The figures for 1920 are shown in Figure 6 and Table III, columns 2 and 3. In the most actively reproducing age group, 20–40 years, the total white female population shows an excess of 1.4 in 32.5 or about 4.3 per cent over the native white female population.

This accounts, no doubt, in part, for the excess of the observed over the calculated birthrate and for the corresponding deficiency in the observed deathrate.

(b) The second mode of approach to the problem of the influence of immigration upon the birthrate is that adopted by Professor J. M. Gillette, in a study published in this JOURNAL, 1916, pages 345–380. We shall not follow his method here, partly because he has already exemplified it sufficiently, and partly because the abrupt change in immigration conditions about 1915 makes its application to the period here considered exceptionally difficult and unreliable.

However that may be, the excess of the observed over the calculated birthrate is some 11 per cent of the observed, so that only a relatively small part of this is so far accounted for by the immediate effect of immigration and we must now examine the second source of divergence between the observed actual rates and the calculated "standardized" rates. This second source is to be found in the steady decrease of the birthrate in past years.

TABLE III

COMPARISON OF TOTAL, NATIVE, AND "STANDARD" POPULATION (WHITE FEMALES)

Maternity frequency per annum and mortality as of 1920

Age distribution in reproductive period, and resulting births

Age group	Age distribution				Births		
	Total white females $N(a)$ Age group as per cent of all ages	Native white females $N'(a)$ Age group as per cent of all ages	"Standard" population $N''(a)$ * White females as per cent of all ages	Maternity frequency per annum $m(a)$ per 100,000 of all marital states	Total white product $N\,m(a)$ = births (daughters)	Native white product $N'\,m(a)$ = births (daughters)	"Standard" product $N''\,m(a)$ = births (daughters)
(1)	(2)	(3)	(4)	(5)	(6) = (2)×(5)	(7) = (3)×(5)	(8) = (4)×(5)
10–14	10.00	11.13	8.65	9	90	100	78
15–19	9.00	9.72	8.31	2,202	19,818	21,403	18,299
20–24	8.99	9.20	7.91	7,310	65,717	67,252	57,822
25–29	8.73	8.43	7.47	7,480	65,300	63,056	55,876
30–34	7.69	7.11	7.02	5,780	44,448	44,448	40,633
35–39	7.12	6.40	6.59	3,898	27,754	24,947	25,688
40–44	5.97	5.34	6.15	1,552	9,265	8,288	9,483
45–49	5.20	4.61	5.74	172	894	793	979
50–54	4.37	3.75	5.30	5	22	19	27
	67.07	65.69	63.06	28,408 ×5			
				142,040			
Total daughters per 10 million females					233,308	230,306	208,885
Per 1,000 females					23.33†	23.03	20.89
Children per 1,000 persons					23.56	23.26	21.34

Note: The figures shown as totals are the daughters born per ten million female population. To find the children of both sexes born per person of both sexes multiply by 2.060 to correct for sex ratio at birth, and divide by the sex ratio in life (2.040‡ for first two, 2.017 for last case). Thus

$$23.33\times\frac{2.060}{2.040}=23.56 \qquad 23.03\times\frac{2.060}{2.040}=23.26 \qquad 20.89\times\frac{2.060}{2.017}=21.34.$$

* Fixed or stable age distribution corresponding to 1920 conditions of maternity frequency and mortality.

† This figure for the United States differs very slightly from the 23.4 for the birth registration area.

‡ Ratio 2.040 from United States 14th Census, 1920, Vol. II, p. 155.

For, by a seeming paradox, a falling birthrate tends reflexly to increase the birthrate for the time being. Though this may at first sight appear almost self-contradictory, it is after all only another instance of a law of inertia such as that which the student of the physical sciences encounters in a variety of different phenomena, and designates, according to circumstances, either as Lenz's Law of self-induction, or as the Principle of Maupertuis, of Le Chatelier, etc. It is in fact a

FIG. 6

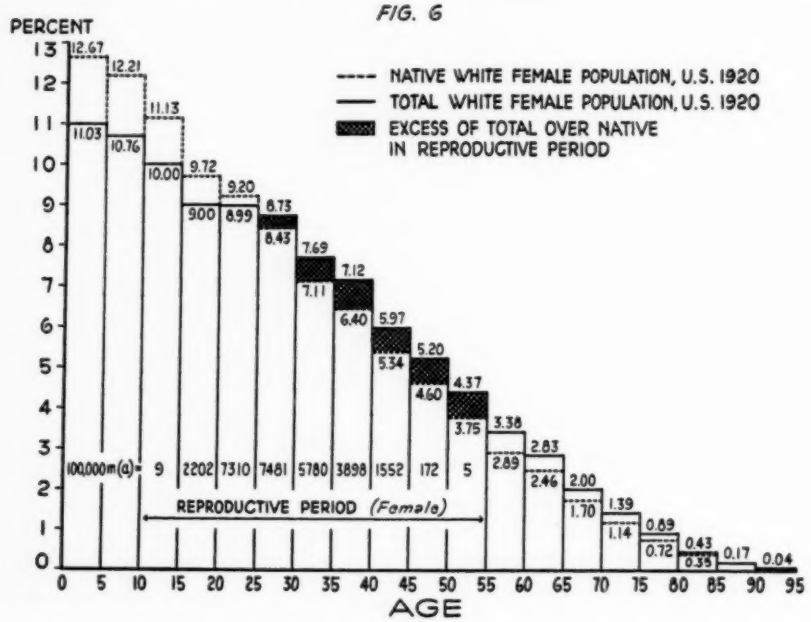


FIGURE 6

COMPARISON OF AGE DISTRIBUTION OF TOTAL AND NATIVE WHITE FEMALE POPULATION UNITED STATES, 1920

Showing excess (shaded) of total over native at child-bearing ages. Maternity frequencies (counting daughters only) indicated by 100,000 $m(a)$.

very general observation that the starting of a given movement automatically evokes influences opposing that movement. Even the social worker may have become familiar with manifestations of this principle.

How this principle operates in the case of a falling birthrate we shall now see from the numerical example before us.

2. *Effect of Steadily Falling Birthrate.* Of the native population, those persons who are today 20 years old, for example, are the survivors of the number who were born 20 years ago, if we disregard the relatively small item of emigration of natives out of the United States.

Thus if we knew the birthrate in past years and had a life table valid for the entire past period under consideration, we could compute the age distribution of the native population today. We could then assure ourselves of the relation between the falling birthrate in past decades on the one hand, and, on the other, the excess of persons of reproductive age in our population.

As a matter of fact, the birthrate in the United States did not become a subject of official record until 1915, so that we cannot carry out this computation.

Retrospective Reconstruction of American Birthrates. We can, however, invert the process, and inquire what kind of a birthrate, in past years, would have resulted in the present native population. This computation has been carried out, as regards males and females separately, with the results shown in Table IV and Figure 7 below. In this computation the population in a given age group was considered as centered in the middle of the group, and the corresponding value of l_x was taken, using a life table, or a combination of life tables, as nearly as possible appropriate to the period to which the figures relate. (See Appendix page 338.)

TABLE IV

RETROSPECTIVE TABLE OF BIRTHRATES (PER 1,000), WHITE

United States observed 1922–1915; computed * 1912–1872
England and Wales observed 1924–1871

		U. S. Unsmoothed	U. S. † Smoothed	Δ	England and Wales		
					Unsmoothed	Smoothed ‡	
U. S. Observed	1924	....	....	....	19.0§	15.1	
	1923	22.2‖	21.8	+ .4	19.7	16.1	
	1922	22.2	22.4	− .2	20.4	17.1	
	1921	24.0	23.0	+1.0	22.4	18.1	
	1920	23.5	23.5	− .5	25.5	19.0	
	1919	22.1	24.1	−2.0	18.5	19.8	
	1918	24.6	24.5	+ .1	17.7	20.6	War years
	1917	24.7	25.0	− .3	17.8	21.3	
	1916	25.0	25.4	− .4	20.9	22.0	
	1915	25.1	25.8	− .7	21.9	22.7	Δ
U. S. Computed	1912	28.1	26.8	+1.3	23.9	24.4	−.5
	1907	28.3	28.2	+ .1	26.5	26.7	−.2
	1902	28.0	29.2	−1.2	28.5	28.3	+.2
	1897	29.4	30.0	− .6	29.6	29.5	+.1
	1892	31.4	30.7	+ .7	30.4	30.7	−.3
	1887	31.4	31.7	− .3	31.9	32.0	−.1
	1882	34.1	33.0	+1.1	33.8	33.6	+.2
	1877	34.0	34.8	− .8	36.0	35.9	+.1
	1876	....	....	....	36.3	36.5	−.2
	1872	37.3	37.2	+ .1	35.6	....	....
	1871	....	....	....	35.0	....	....

* From age distribution 1920 and life table data.
† By third degree parabola based on eleven quinquennial points, 1872–1922. See Figure 7.
‡ By third degree parabola based on 39 points, 1876–1914. See Figure 7.
§ Newspaper report.
‖ This figure includes Negroes. For 1922 the corresponding figure was 22.5.

FIG. 7

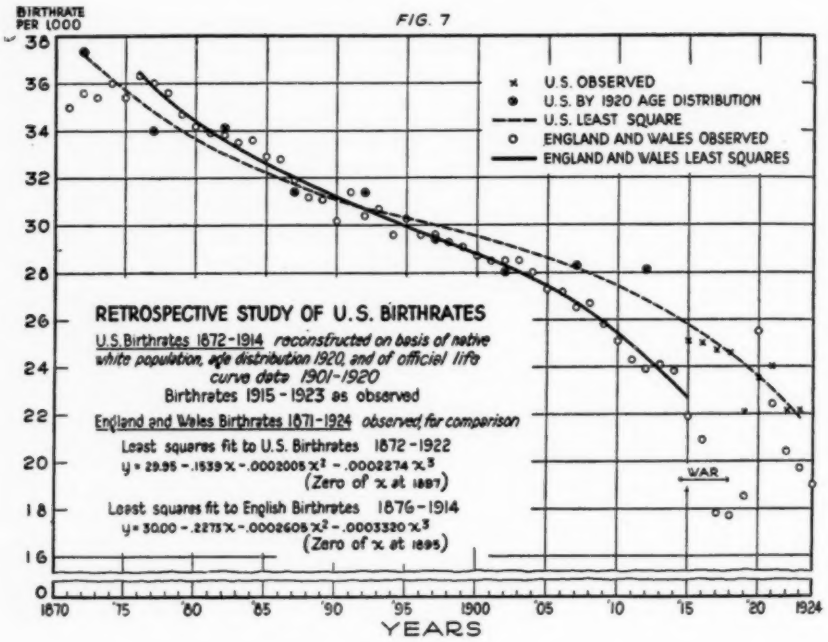


FIGURE 7

RECONSTRUCTION OF UNITED STATES BIRTHRATES 1872–1914

And English observed birthrates for comparison.

The figures thus computed for past decades, combined with the observed figures of more recent years, were fitted by least squares with a cubic parabola

$$y = 29.95 - .1539x - .0002005x^2 - .0002274x^3 \text{ (zero of } x \text{ at 1897).}$$

Considering the range of time covered—a little over half a century—the fit of this simple equation seems noteworthy, especially when viewed in connection with the exceedingly close fit of a similar cubic equation

$$y = 30.00 - .2273x - .0002605x^2 - .0003320x^3 \text{ (zero of } x \text{ at 1895).}$$

to the observed birthrate curve for England and Wales over the period of 39 years, 1876–1914, as shown in Table IV. The war, of course, destroyed the regularity in England, though it had little effect upon it in the United States.

While it is not intended that the figures thus obtained for the United States should be regarded as rendering an accurate account of the birthrates for the years indicated, the general parallelism of the com-

puted United States series and the observed English figures leaves little doubt in one's mind but that the former do truly indicate the general trend of the birthrate in the past.

It is thus seen that a birthrate altogether in keeping with what we may expect to have been prevalent in the past does account for our present age distribution. And that, in turn, our present age distribution exhibits a marked excess, in mid-life, over the "stable" age distribution, is seen in Table III, columns 2, 3, 4, and in Figures 8 and 9.

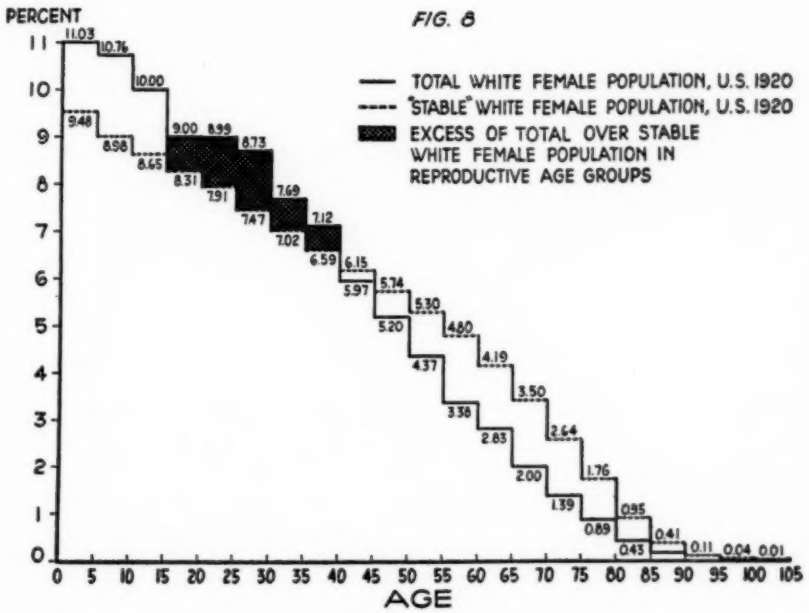


FIGURE 8

COMPARISON OF TOTAL WHITE FEMALE POPULATION UNITED STATES 1920 WITH POPULATION IN STABLE AGE DISTRIBUTION

Shaded areas represent excess of total observed over "stable" population in reproductive age groups.

The term "stable" age distribution is here to be understood as denoting that fixed age distribution which would ultimately become established if our present schedule of maternity for females of different ages, and our present age schedule of mortality, were to continue sufficiently long to allow the age distribution to become adjusted to these existing conditions, so that the reflex swelling of our present birthrate through the hang-over effect of the past decline should no longer be felt.

Check upon Computation of Standard Population. That the age distribution computed for the given demographic features (age schedule

of maternity frequency and of mortality) does actually and consistently give rise to the computed birthrate is easily controlled by applying to the age groups in the reproductive periods, in the computed standard (stable) population, the factors of the given maternity schedule. The pertinent data will be found in Table III, column 8, and, as will be seen, the agreement is essentially perfect. Thus, for example, the birthrate computed for the stable or standard population directly from the value of r given above (5.47 per thousand per annum),

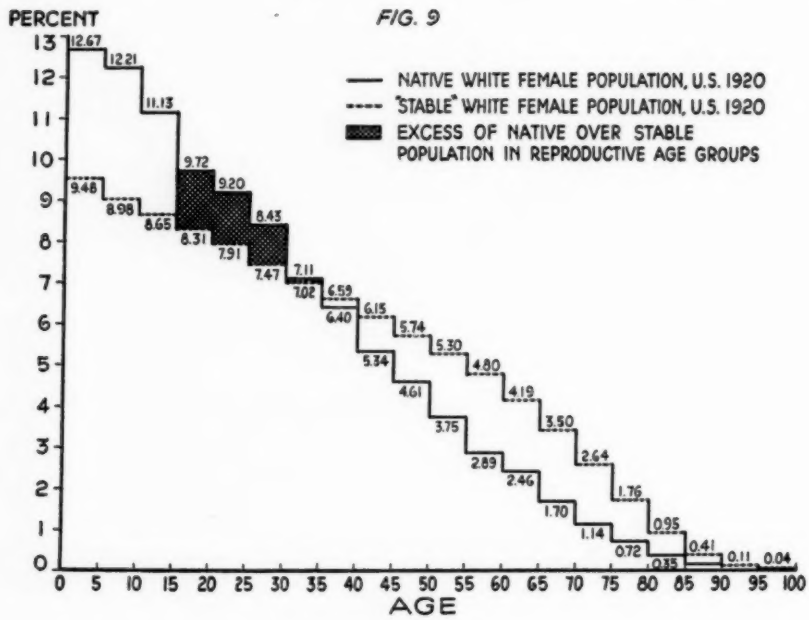

PERCENT
FIG. 9
NATIVE WHITE FEMALE POPULATION, U.S. 1920
"STABLE" WHITE FEMALE POPULATION, U.S. 1920
EXCESS OF NATIVE OVER STABLE POPULATION IN REPRODUCTIVE AGE GROUPS
AGE

FIGURE 9

COMPARISON OF NATIVE WHITE FEMALE POPULATION UNITED STATES 1920 WITH POPULATION IN STABLE AGE DISTRIBUTION

Shaded areas represent excess of native observed over "stable" population in reproductive age groups.

is 20.90; compare with this figure the value 20.89 found from the age distribution in the reproductive period in the standard population and the given maternity frequency as shown at the foot of the last column in Table III. This agreement checks not only our computation but also the argument underlying it. The graphic interpretation of this agreement is indicated in Figure 10 which exhibits the age distribution in the female standard population. The shaded area represents that portion of the population which is reproducing, and the

integrated product of the elements of this area by the corresponding maternity frequencies (for births of daughters) just gives the zero ordinate, which, of course, represents the birthrate (daughters per female per annum). This relation between the shaded area and the zero ordinate exists in *any* population; but the characteristic feature of the

FIG. 10

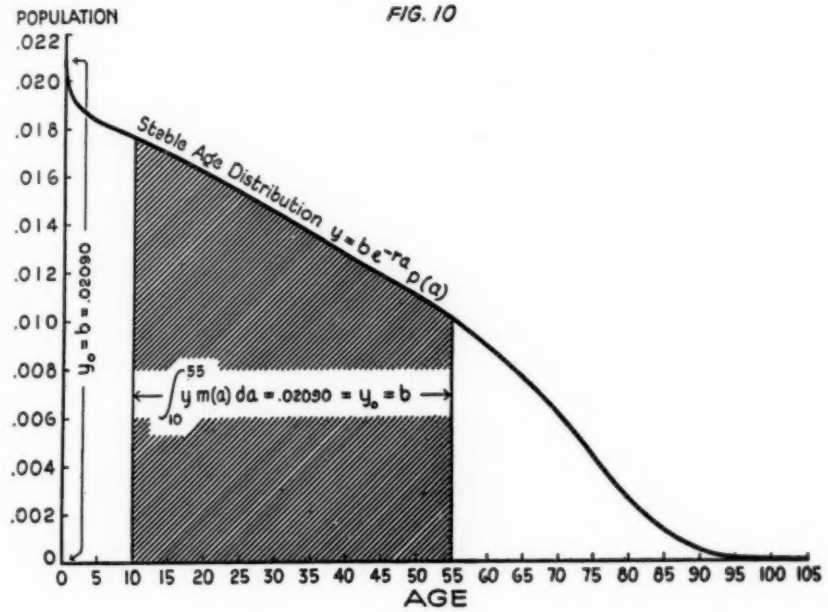


FIGURE 10

ILLUSTRATION OF RELATION BETWEEN ZERO ORDINATE (BIRTHRATE) AND AREA IN REPRODUCTIVE AGE GROUPS (SHADED AREA IN FIGURE)

In "stable" age distribution curve of female population United States 1920. The curve is drawn for one unit of population so that its total area is unity.

"stable" [1] standard here computed is that its form is fixed, it is the unique age distribution permanently compatible with the 1920 age schedule of maternity frequency and mortality.

[1] Perhaps it may be well, in order to prevent misunderstanding, to point out that the "stable" or "standardized" population here used as a standard of reference is not a "stationary" population. A stationary population is not compatible with the age schedule of maternity frequency and mortality as of 1920, whereas the standard here employed is not only compatible with these conditions, but is precisely the distribution of ages to which these conditions, if permanently maintained, would eventually lead. Furthermore, the standard here employed is not a mere arbitrary distribution of ages, such as commonly used for purposes of comparison in vital statistics, but is such as to represent the inherent reproductive and vital properties of the population as it actually existed in 1920. Only the adventitious and disturbing factor of the variable age distribution, a temporary condition referable to the past history of the population, is eliminated in correcting to this standard of the fixed age distribution corresponding to the permanent establishment of the existing maternity frequencies and mortalities at the several ages of life.

The figures in Table III, column 6, further furnish a check [1] upon the birthrate, namely, 23.33 daughters per female, 23.56 children per person, computed; as against 23.4 and 23.5 observed. For explanation of the factor to convert birthrates of daughters per female to children per person see the Appendix.

It must now be clear how our present excess over the normal, of persons in mid-life, leads to a birthrate in excess of that computed as normally corresponding to our age schedule of maternity and mortality. We are, as it were, living upon our past. What maintains, for the time being, a high birthrate, is not our inherently high reproductive vitality, but the fact that the surviving descendants of a more highly reproductive generation are swelling the ranks of middle life, and lending a spurious appearance of vigorous growth to an otherwise rather meagerly reproducing population. We must realize that, if the present rate of procreation should persist, ultimately the excess of persons in early mid-life would disappear, and then we should be faced, not with the present birthrate [2] of .02340, but with a birthrate of only .02090. At the same time the deathrate, instead of .01241, would assume the value .01543. And what is most striking of all, while the lowering of the birthrate would be only a little over 10 per cent and the rise in the deathrate only about 23 per cent, the effect upon the rate of increase, $(b-d)$, would be to reduce it from .01099 to .00547, that is to say, to reduce it to 50 per cent of its present value.

EFFECT OF MODIFICATIONS IN DEMOGRAPHIC FACTORS UPON RATE OF INCREASE

Effect of Improvement in Mortality. An interesting comparative study which suggests itself in connection with our present considerations is the inquiry into the particular share which recent improvements in the general public health, as expressed by the figures of our life tables, have had in counterbalancing the diminishing reproductive activity of our population. To make the question concrete, let us ask: What would be the rate of increase of our population, aside from immigration (and emigration), if our present reproductive habitus were associated, not with the life table of 1920, but with that of 1910 or of 1901? The answer, the results of computations carried out on the basis indicated, will be found in Table V. It will be seen that the life table of 1910 would leave us with a small margin, a rate of natural increase of 3.59 per thousand per annum; the 1901 life table would barely keep us from the downgrade, with a very small, but still positive rate of

[1] Purely arithmetical in this case.

[2] Reference should again be made to footnote (1), p. 305.

TABLE V

NATURAL RATE OF INCREASE, UNITED STATES, 1920, WHITE POPULATION

Reproductive cycle Maternity frequency	Life table $p(a)$	Natural rate of increase		Daughters per 1,000 females (per entire lifetime)			§Children (both sexes) per mother	Length of generation (years)	Birthrate b per 1,000 per annum				Deathrate d per 1,000 per annum		Natural rate of increase $b-d=r$ per 1,000 per annum	
									Computed for standard age distribution‖		Observed with actual age distribution					
		Per annum	Per generation	All females*	Married females†	Mothers‡			Daughters per female	Children per person (both sexes)	Daughters per female	Children per person (both sexes)	Computed for standard age distribution	Observed with actual age distribution	Computed for standard age distribution	Observed with actual age distribution
		1,000 r	1,000 R													
(1)	(2)	(3)	(3a)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
U. S.; B. R. A. 1920	1920	5.47	1,168	1,168	1,488	1,786	3.68	28.3	20.90	21.34	23.4	23.5	15.43	12.41	5.47	10.99
Same	1910	3.59	1,107	1,107	1,490	1,790	3.68	28.4	21.04	.....	.....	....	17.45	.....	3.59	.....
Same	1901	1.92	1,056	1,056	1,477	1,775	3.66	28.4	20.88	.....	.....	....	18.96	.....	1.92	.....
Same, but shifted 5 years forward	1920	3.62	1,128	1,128	1,437	1,724	3.55	33.4	19.68	.....	.....	....	16.06	.....	3.62	.....
U. S.; B. R. A. 1920	No deaths in reproductive period¶	12.32	1,420	1,420	1,809	2,171	4.47	28.5	24.27	.....	.....	....	11.95	.....	12.32	.....

* These figures are based on 1,000 women born, not on 1,000 women living.

† These figures are based on 1,274 females born, of which 1,000 marry, and 274 die single, according to United States Census, 1920, Population, Vol. II, p. 392.

‡ The figures for 1920 are based on 1,529 females born, of which 1,000 become mothers and 529 die childless, according to United States Census, 1920, Population, Vol. II, p. 392, assuming that one-sixth of all married women die childless; the figures for 1910 and 1901 are on a compounding basis.

§ On the basis of a sex ratio of 1.06 male births to 1.00 female births.

‖ The term "standard" age distribution denotes that age distribution which would ultimately become established if the existing values of $m(a)$ and $p(a)$ remained unchanged for a long period of time.

¶ All deaths assumed to take place exactly at mean length of life, *viz.*, 57.52 years of age.

increase of 1.92 per thousand per annum. The same facts are also brought out in another way in the following table, which gives, for the several conditions here considered, the time which would be required for the population to double:

	Time of doubling years
With observed rate of increase as of 1920	63
With ultimate "standardized" rate of increase:	
(a) Maternity as of 1920, mortality as of 1920	127
(b) Maternity as of 1920 but postponed 5 years, mortality as of 1920	190
(c) Maternity as of 1920, mortality as of 1910	192
(d) Maternity as of 1920, mortality as of 1901	361

Effect of Postponement of Marriage. An exact discussion of the effect of postponement (or advancement) of marriage, upon the natural rate of increase, cannot be given on the present occasion. But there is one special case which can very easily be treated. Suppose the reproductive cycle were shifted forward (postponed to later ages) bodily, so that, for example, a female of age 25 reproduced on an average at the present rate characteristic of age 20, and so on all the way along the scale (see Figure 2). The resulting natural rate of increase can obviously be computed by the same method as for the actual condition. Carrying out the computation (by the exact method) we find the following values

$$b = .0196$$
$$d = .0160$$
$$r = .0036$$

Thus it is seen that the postponement of the reproductive cycle by five years diminishes the natural rate of increase of the population by about the same amount as a turning back of the clock by ten years as regards the mortality figures given in the life table.

It may be remarked in passing that the diminution of the rate of increase by a postponement of marriage is due in part to the increase of the mean length of one generation occasioned by the postponement, and in part to the diminished survivals to marriage age when that age is increased. A formula which brings out this fact will be found in the Appendix.

SIZE OF FAMILY

The term "size of family" is notoriously indefinite. It is unnecessary here to point out the uncertainty of its interpretation. For our present purpose it will be better to deal with the closely related and more definite concept "average number of children per mother." From this the "size of a normal family" may be obtained by adding 2, the parents completing the family.

Number of Children per Mother. We have incidentally found the number of daughters born to a cohort of 1,000 females entering life together. This number, according to maternity and mortality conditions of 1920, was 1,168. With a rate of 1.06 males to 1 female birth, this gives 1.238 sons, or 2.406 children per female, counted regardless of marital condition. However, of the cohort of 1,000 females starting out upon life together, only 785 ultimately marry, the other 215 dying single, either before reaching puberty, or at adult ages. Thus, if we neglect the relatively unimportant item of illegitimate births, 2,406 children are born by 785 married women, giving an average of 3.06 children per married woman. If we further reduce our basis of reference to mothers, that is, to married women having one or more children, assuming one marriage out of six [1] to be childless (either on account of the death of wife or husband soon after marriage, or on account of sterility, natural or otherwise), we find for the number of children per mother the figure 3.68, as indicated in Table V, column 7.

Corresponding figures, also shown in Table V, have been calculated for the following additional cases:

1. Age schedule of maternity frequency as of 1920 associated with mortality as of 1910.
2. The same, associated with mortality as of 1901.
3. The same, associated with "no deaths" from age zero to end of reproductive period.
4. Age schedule of maternity frequency as of 1920, but postponed five years throughout, associated with mortality as of 1920.

Case 3 is hypothetical and represents a case that could not be realized in practice, but is of interest as showing the degree of the influence of deaths, up to the end of the reproductive period, in depressing the ratio between two successive generations from its potential maximum of 1.420 to the actual value 1.168. The figures thus stated, are perhaps a little deceiving, as they do not suggest to the eye a very marked difference. But in the course of three generations the progeny, for the case of "no deaths" to end of reproductive period, would bear to the progeny with deaths as of 1920, the ratio $\left(\frac{1.420}{1.168}\right)^3 = 1.795$. Thus in the course of three generations, say 90 years, the deaths of potential mothers cause a loss equal to four-fifths of the actual progeny.

Comparison with Stationary Population. A comparison of some interest is that of the number of children per mother computed as

[1] For a résumé of investigations regarding the percentage of marriages that remain sterile, see Reynolds and Macomber, *Fertility and Sterility in Human Marriage*, Saunders, 1924, pp. 28–33.

above, and corresponding to the present state (1920) of the age schedule of maternity frequency and to the present life table (1920), contrasted with the number of children per mother required to maintain an exactly stationary population. To obtain this last figure we consider a cohort of 1,000 females born upon a given day. *For equilibrium* these must ultimately have 1,000 daughters. From the life table we know how many of these 1,000 females die each year, and from census data we know how many of them die single. The pertinent figures are exhibited in Table VI. It is thus found that, according to maternity frequency and mortality conditions of 1920, a residue of 215, out of 1,000 born,

TABLE VI

PROPORTION OF PERSONS DYING SINGLE AT DIFFERENT AGE PERIODS AND THROUGHOUT LIFE, OUT OF 100,000 BORN

Age	Males			Females		
	Deaths	Single per cent	Deaths, single	Deaths	Single per cent	Deaths, single
Under 15	14,250	100.0	14,250	11,950	100.0	11,950
15	273	99.8	272	240	98.6	237
16	307	99.7	306	270	95.7	258
17	335	99.1	332	302	89.9	271
18	358	96.9	347	340	80.0	272
19	376	93.0	350	377	70.3	265
20	390	86.8	339	410	60.0	246
21	402	78.2	314	440	52.5	231
22	412	70.6	291	465	44.9	209
23	425	63.0	268	486	38.3	186
24	436	56.3	245	505	33.0	167
25	447	49.6	222	520	29.0	151
26	458	44.1	202	532	25.3	135
27	470	38.4	181	543	22.0	119
28	485	34.6	168	550	20.2	111
29	499	29.5	147	555	17.5	97
30	514	29.0	149	560	18.5	104
31	527	24.7	130	563	14.5	81
32	540	24.4	132	564	14.7	83
33	552	21.5	119	563	12.7	71
34	560	20.2	113	562	12.8	72
35–44	5,975	16.1	962	5,650	11.4	644
45–54	8,400	12.0	1,008	7,800	9.6	749
55–64	13,675	9.8	1,340	12,800	8.4	1,075
65 and over	49,075	7.3	3,582	52,600	7.1	3,735
Total			25,769			21,519

SONS, DAUGHTERS AND CHILDREN PER MARRIED PERSON

(1) *Based on males*

1 − .25769 = .74231
1/.47321 = 1.34715 sons, per married male
1.34715 = 1.27089 daughters, per married male
1.06
2.61804 children, per married male

(2) *Based on females*

1 − .21522 = .74878
1/.74878 = 1.27419 daughters, per married female
1.27419 × 1.06 = 1.35064 sons, per married female
2.62483 children, per married female

(3) *Sources*

1. Deaths * according to Foudray U. S. Abridged Life Table, 1919–1920, pp. 20–21 (males); pp. 22–23 (females).
2. Marital Condition, According to 14th Census of U. S., 1920, Vol. II, pp. 391–392.

* Graphic interpolation.

die single, and 785 marry. These 785 must, then, have 1,000 daughters, or 1,000 must have 1,274 daughters. At a ratio of 1.06 male to 1 female birth, the 1,000 females will have 1,350 sons. This result received an independent check by a separate computation of the number of sons per male required to maintain the population stationary. As will be seen from Table VI the agreement is exceedingly close.

Thus, *for equilibrium*, every married woman must have, on an average, $1.27+1.35=2.62$ children. But as already pointed out, not all married women become mothers. Some die without issue in the early years of their marriage; other are sterile and have no children though they live through the entire reproductive period. It seems difficult to get any exact estimate of the number of married women that die childless. If we estimate as before the proportion of childless marriages at one-sixth then five-sixths of the 785 married women, that is to say 654 out of 1,000 females born, must have 1,000 daughters, or, on an average a mother must have 3.15 children.

This figure may appear surprisingly high. It bears comparison with the several figures shown in Table V, column 7, where the corresponding figure for the actual number of children per mother, under several stated conditions is shown.

Comparison with Number of Children per Mother Living. It may be well to put in a word of caution. The number of children per mother, computed as an average for 1,000 females born, is not the same figure as the number of children per mother *in the living population.* It must be remembered that there are a variety of different ways of taking an average. The distinction which we have to make here is somewhat similar to that between the mean length of life and the average age of the living. These two are not the same except in a stationary population.

The observed figure for the average number of children per mother living has been remarkably constant for some years past, according to the United States Birth Statistics, 1922, p. 23; 1921, p. 19; 1920, p. 15, namely, 3.3 (for all races) in 1920, 1921 and 1922. For white mothers the figure for 1922 is 3.2.

SUMMARY

The natural rate of increase of a population, if measured directly by the excess of the birthrate over the deathrate, gives a misleading impression of its reproductive vitality owing to the interfering effect of an age distribution conditioned by the previous history of the population. In the United States heavy immigration and a rapidly declining birthrate have increased disproportionately the population at the child-

bearing ages. A method has been described and illustrated by numerical example for obtaining a true measure of the natural rate of increase of a population, having given its biological constitution as regards the age schedule of maternity frequency and mortality. Applying this method to the population of the United States as constituted according to the Census of 1920, the following results are obtained:

1. In place of the actual rate of natural increase of 10.99 per thousand per annum, the biological constitution (age schedule of maternity frequency and mortality) would give us a rate of only 5.47 per thousand as soon as the influence of immigration and decline of birthrate in the past had had time to wear off, so that we should no longer have a disporportionately high population in the reproductive age group.

A number of results of independent interest were obtained as by-products of the main investigation, as follows:

2. A precise measure of the mean length of generation was found. Reckoning from mother to daughter its value (1920) is 28.33 years; from father to son 32.98 years. The difference between these two figures is due chiefly to the later age of males at marriage; in lesser degree to the somewhat diminished survival of the males to that later age.

3. The true or standardized rate of natural increase of 5.47 per thousand per annum defined by the biological constitution of the population, is the resultant of a birthrate of 20.9 and a deathrate of 15.4. These would replace the observed birthrate of 23.4 and deathrate of 10.99 when the age distribution had become normally adjusted to the biological constitution of the population.

4. A retrospectively constructed series of birthrates in the United States 1870–1915 has been established on the basis of the native population regarded as survivors of past births. For details see Table V.

5. A study of the influence of improvement in mortality rates upon the true natural rate of increase of the population shows that the present (1920) fecundity of females associated with the mortality of 1910 and 1901 would leave a residual natural rate of increase of only 3.59 and 1.92 per thousand per annum respectively. The corresponding periods for a doubling of the population are 127, 192 and 361 years respectively, with mortalities as of 1920, 1910 and 1901.

6. A postponement of the female reproductive cycle by five years would reduce the rates of natural increase just about as much as a reversion to 1910 mortality rates.

7. The number of children per married female corresponding to the biological constitution of the population of 1920 is 3.06. On the basis of one-sixth of all marriages being without issue this gives 3.67 children

per fertile married female. Adding to this the father and mother gives the numbers 5.06 and 5.67 respectively for the complete average family.

8. The corresponding numbers of children required in order just to maintain a stationary population are 2.62 and 3.15, when referred to all married women or fertile (five-sixths of all) married women respectively.

Bibliographic Note

The following references bearing on one point or another of the subject here discussed have been noted:

A. L. Bowley, "Births and Population in Great Britain," *Economics Journal*, June, 1924, p. 191.

J. Brownlee, "Birth Control in Some of its Statistical Aspects," *Lancet*, November 1, 1924, p. 927.

A. M. Carr Saunders, "Human Fecundity," *The Population Problem*, p. 103.

F. Savorgnan, "La fecondita della aristocrazie," *Metron*, 1924, vol. 3, p. 468.

C. T. Brunner, "Local Variations in the Birth-Rate," *Economic Journal*, March, 1925, p. 60.

APPENDIX [1]

RIGOROUS ARGUMENT AND METHODS OF COMPUTATION

1. *The Demographic Functions of a Population with Stable Age Distribution.* The stable age distribution which must ultimately become established in a population with fixed age schedule of maternity and mortality is given by the system of equations:

$$c(a) = be^{-ra}\ p(a) \tag{1}$$

$$\frac{1}{b} = \int_o^\infty e^{-ra}\ p(a)\ da \tag{2}$$

$$1 = \int_o^\infty e^{-ra}\ p(a)\ m(a)\ da \tag{3}$$

in which the several symbols have the following meaning:

$c(a)$ is the coefficient of age distribution, such that, out of a total number N of persons, a number $N\ c(a)\ da$ are comprised within the age limits a and $a+da$.

b is the birthrate per head per annum.

r is the "natural rate of increase," the excess of the birthrate b over the deathrate d.

$p(a)$ is the probability, at birth, that a random individual will reach age a. It is the quantity tabulated as l_x in a life table, if we interpret this on the supposition that the radix of the life table is unity.

[1] Prepared by A. J. Lotka.

$m(a)$ is the maternity frequency per annum of females of age a.

For the derivation of these formulae,[1] and the proof that the age distribution indicated ultimately becomes established under the conditions stated, the reader must be referred to the literature.[2] Here it must suffice to show how these formulae lead, on the one hand, to the computation of the exact values of the several quantities cited in the text; and, on the other hand, how we may derive from them the approximate formulae there given without rigorous proof.

The Natural Rate of Increase. Equation (3) determines r, the natural rate of increase in the population with stable age distribution, when $p(a)$ and $m(a)$ are given, that is to say, when a sufficiently detailed life table and statistical record of maternity frequency at different ages is available.

In the present instance the United States Abridged Life Tables 1919–20 (E. Foudray), for this period, gave the values of $p(a)$ (*i.e.* l_x) at ages 0, 1, 2, 7, 12, 17, . . . 92 and the values of q_x at the same ages. From these the values of $p(a)$ (*i.e.* l_x) can by the usual actuarial method be obtained for ages 12.5, 17.5, . . . 52.5. These values, as will appear presently, are needed in our computations. It was found unnecessary to have detailed values for every year of age. The figures for the white females in the "aggregate" of all states covered in these tables were used in the present study.

For 1910 and 1901 the Glover life tables furnish values of $p(a)$ for every year of life directly.

Values of $m(a)$ blocked together in five-year groups are found in the *Sixth Annual Report of Birth Statistics* (1920, published in 1922), pages 168, *et seq.* They cover 24 states, of which one, however (Maine), reports only total births. The remaining 23 states, therefore, form the basis of this study. The figures referring to white females were used, interest being, for obvious reasons, principally centered in the white races, while the data for females were selected for the reasons indicated in the text p. 307.

Method of Computation of r. Equation (3) is an implicit expression for r, and cannot be solved directly by any arithmetical process.

We can, however, proceed as follows: Let us denote the integral of the right hand member of (3) by y, so that, $p(a)$ and $m(a)$ being given, y is a function of r. Differentiating with respect to r we have

[1] It is to be remarked that the formulae (1), (2), (3) may be applied to either sex separately, or to both sexes combined, for ultimately the two sexes must increase at the same rate r, otherwise the ratio between them would eventually exceed all bounds.

[2] *Phil. Mag.*, April, 1911, p. 435; *Proc. Natl. Acad. Science*, 1922, Vol. 8, p. 339; *Jl. Washington Acad. Science*, 1913, Vol. 3, pp. 241, 289; *Amer. Jl. Science*, 1907, Vol. 24, p. 199; This JOURNAL, 1918, p. 121; 1921, p. 997.

$$\frac{dy}{dr} = -\int_o^\infty ae^{-ra} \quad p(a)\, m(a)\, da \tag{4}$$

$$= -A\int_o^\infty e^{-ra} \quad p(a)\, m(a)\, da \tag{5}$$

$$= -Ay \tag{6}$$

Integrating we have

$$y = y_o e^{-\int A\, dr} \tag{7}$$

Now when $r = 0$

$$y = y_o = \int_o^\infty p(a)\, m(a)\, da \tag{8}$$

But this is the quantity which we have, in the text, denoted by R_o, the ratio between the total births in two successive generations. Hence we have

$$y = R_o e^{-\int A\, dr} \tag{9}$$

We seek a value of r such that y is equal to unity. This value, then, is given by

$$1 = R_o e^{-\int A\, dr} \tag{10}$$

or

$$R_o = e^{\int A\, dr} \tag{11}$$

It remains to determine numerically the value of the exponent $\int A\, dr$. The definition of A is, in accordance with (4), (5)

$$A = \frac{\int_o^\infty ae^{-ra}\, p(a)\, m(a)\, da}{\int_o^\infty e^{-ra}\, p(a)\, m(a)\, da} \tag{12}$$

Expanding e^{-ra} by Taylor's theorem and writing for brevity

$$R_n = \int_o^\infty a^n\, p(a)\, m(a)\, da \tag{13}$$

we find

$$A = \frac{R_1 - r\,R_2 + \frac{r^2}{2}R_3 - \frac{r^3}{6}R_4 + \ldots}{R_o - r\,R_1 + \frac{r^2}{2}R_2 - \frac{r^3}{6}R_3 + \ldots} \tag{14}$$

$$= \alpha + \beta r + \gamma r^2 + \delta r^3 + \ldots \tag{15}$$

where

$$\alpha = \frac{R_1}{R_o} \tag{16}$$

$$\beta = \alpha^2 - \frac{R_2}{R_o} \tag{17}$$

$$\gamma = \alpha^3 - \frac{3}{2}\alpha\frac{R_2}{R_o} + \frac{1}{2}\frac{R_3}{R_o} \tag{18}$$

$$\delta = \alpha^4 - \alpha^2\frac{R_2}{R_o} - \frac{1}{3}\alpha\frac{R_3}{R_o} + \frac{1}{2}\frac{R_2^2}{R_o} - \frac{1}{6}\frac{R_4}{R_o} \tag{19}$$

It is to be particularly noted that the series (15) converges very rapidly,[1] owing to the fact that the constants R follow approximately the relation

$$\frac{R_n + 1}{R_n} = \text{const} = \frac{R_1}{R_o} \tag{20}$$

and the sum of the numerical coefficients of the several terms in the expressions for β, γ, δ, . . . is zero, while the substitution

$$R_n = R_o\left(\frac{R_1}{R_o}\right)^n \tag{21}$$

converts each of the literal coefficients in such an expression into the same power of α, so that the expression as a whole vanishes. Actually the relation (20) holds only approximately, and the coefficients β, γ, . . . do not vanish, but are small.

In actual fact, as shown below, the terms beyond βr are negligible, so that we have, finally

$$\int A\ dr = \alpha r + \tfrac{1}{2}\beta r^2 \tag{22}$$

and

$$R_o = e^{\alpha r + \frac{1}{2}\beta r^2} \tag{23}$$

$$\tfrac{1}{2}\beta r^2 + \alpha r - \log_e R_o = 0 \tag{24}$$

a quadratic equation for r.

Numerical Values. Using the Census data for 1920 and the Foudray life table 1919–20 (white females), the following numerical values were obtained.[2]

[1] It is for this reason here developed in preference to the more slowly converging series obtainable directly from (3) by expanding the exponential under the integral sign.

For an alternative method of deriving the coefficients α, β, etc., see Whitacker and Robinson, *Calculus of Observations*, p. 170, equation (4) and the preceding matter.

[2] One special case was worked out using, in the evaluation of the integral (3), a smoothed curve in place of the stepped curve of Fig. 1. It was found that the resulting value of r differed but little from that obtained by the very much more economical use of the stepped curve, and thereafter the latter was used throughout in the computations.

n	R_n	$\frac{R_n}{R_{n-1}}$	Δ
0	1.168	.....	.....
1	33.23	28.450	1.607
2	998.79	30.057	1.563
3	31,581.34	31.620	1.480
4	1,045,342	33.100	

$$\alpha = 28.45 \qquad \log_e R_0 = .1550.$$
$$\beta = -45.71.$$

So that the quadratic equation for r is

$$22.85r^2 - 28.45r + .1550 = 0.$$

And finally

$r = .00547$

$= 5.47$ daughters per 1,000 female population per annum.

The correctness of this value was controlled (as a warrant for neglecting terms of third and high degree in r) by computing the integral

$$\int_0^\infty e^{-ra} p(a)\, m(a)\, da = 1 \tag{3}$$

with $r = .00547$. The value of the integral came out 1.0002.

The relation of the value of r thus found to the approximate value given in the text is seen from the following consideration:

Expanding the integral in (3) by Taylor's theorem we have

$$1 = \int_0^\infty p(a)\, m(a)\, da - r\int_0^\infty ap(a)\, ma\, da + \ldots \tag{25}$$
$$= R_0 - rR_1 + \ldots \tag{26}$$
$$= R_0\left(1 - r\frac{R_1}{R_0}\right) + \ldots \tag{27}$$

Comparison with the text shows that $\frac{R_1}{R_0}$ is the quantity there denoted by T, the "center of gravity" age of the maternity frequency curve.

Thus

$$\left.\begin{aligned} 1 &= R_0(1 - r\,T) \\ R_0 &= 1 + r\,T \text{ if } r \text{ is small} \\ &= (1+r)^T \text{ if } r \text{ is small} \end{aligned}\right\} (28)$$

This is the approximate formula given in the text.

Rate of Increase as Modified by Postponed Marriage. The rate of increase r_n for the special case in which the reproductive cycle is postponed n years may be found directly by the general method; or when r

is known for the case of the cycle in its natural position, the value of r_n is found readily as follows: Let a change in the mean length of one generation from T to T' be accompanied by a change of r to r' and a change of R_o to R_o' Then we have

$$e^{r\,T} = R_o \tag{29}$$

$$e^{r'T'} = R_o' \tag{30}$$

$$\log_e \frac{R_o'}{R_o} = r'T' - r\,T \tag{31}$$

$$r' = \frac{1}{T'}\left(r\;T + \log_e \frac{R_o'}{R_o}\right) \tag{32}$$

A little reflection shows that we may expect $\frac{R_o'}{R_o}$ to be given, in close approximation, by the ratio of $p(T')$ *to* $p(T)$, so that, to this approximation

$$r' = \frac{1}{T'}\left(r\,T + \log_e \frac{p(T')}{pT}\right) \tag{33}$$

The postponement of the reproductive cycle by n years increases T very nearly by n years, so that $T' = T+n$, and, if we denote the corresponding value by r_n, we have accordingly:

$$r_n = \frac{1}{T+n}\left\{ r\,T + \log_e \frac{p(T+n)}{pT} \right\} \tag{34}$$

In the numerical instance here considered, in which $n=5$ and T is in the neighborhood of 30 years of age, it is found that $\frac{p(T+5)}{p(T)}$ is nearly independent of small changes in T and has the value .9664.

Now $\log_e .9664 = -.03417$, so that

$$r_5 = \frac{1}{T+5}(r\;T - .03417) \tag{35}$$

With $r = .00547$ this gives

$$r_5 = .00362.$$

The essential correctness of this relation was established by the fact that the value of r, thus found, when substituted in the integral (3), gave, upon summation, the value unity.

Mean Length of One Generation. Inasmuch as (23) can be written

$$R_o = e^{r(\alpha + \frac{1}{2}\beta r)} \tag{36}$$

it is evident that the mean length of one generation is given by

$$\begin{aligned} T &= \alpha + \tfrac{1}{2}\beta r \\ &= 28.33 \text{ years} \end{aligned} \tag{37}$$

The value 28.45 given in the text is the first approximation obtained by neglecting the second term in the exponent in (36) and writing simply

$$T = \alpha \tag{38}$$

$$= \frac{R_1}{R_0} \tag{39}$$

Birthrate. An argument similar to that developed above with respect to the computation of r according to the equation (3) applies also to the computation of b according to the equation (2). Writing

$$L_n = \int_0^\infty a^n \, p(a) \, da \tag{40}$$

we thus find

$$b = \frac{e^{\int A' dr}}{L_0} \tag{41}$$

where

$$A' = u + vr + wr^2 + \ldots \tag{42}$$

and

$$\left.\begin{aligned} u &= \frac{L_1}{L_2} \\ v &= u^2 - \frac{L_2}{L_0} \\ w &= u^3 - \frac{3}{2}u^2\frac{L_2}{L_0} + \frac{1}{2}\frac{L_3}{L_0} \end{aligned}\right\} \tag{43}$$

The numerical values of the several constants in this case are

	$\frac{L_n}{L_n - 1}$
$L_0 = 57.52$	
$L_1 = 2020$	35.11
$L_2 = 99.102$	49.0
$L_3 = 5{,}653{,}000$	56.7
$L_4 = 350{,}937{,}000$	62.2
$u = 35.11$	
$v = -490.2$	
$w = 1{,}682.8$	

We find, accordingly, by (41), (42) and (43), using for r the value previously determined, namely, .00547,

$$\log_e b \; L_0 = .184816$$

$$b = \frac{1.2030}{57.52}$$

$$= .02091$$

A check upon this computation of b was obtained by direct integration of (2) after substituting therein the known value of r. The integration was carried out (a) arithmetically, using the trapezoid formula for the essentially straight portions of the curve and Simpson's rule for the more curved portions. The integration was also carried out (b) with the aid of the planimeter. The results of these several computations are shown in Table VII below.

Stable Age Distribution. Since r and b are now determined, the stable age distribution follows immediately according to (1). In the computation of the five-year age groups advantage is taken of the integration already performed in determining b according to (2). The results of this computation of the standard age distribution have already been given in the text page 5, and Table I.

Birthrate Computed from Maternity Frequency. The proportion of females in each age group being now given, the birthrate can be determined by the third method (c), which consists in multiplying the female population in each age group by the corresponding maternity frequency. The birthrate so found should, of course, agree with that found by methods (a) and (b) given above. The actual results obtained by the several methods were as follows:

TABLE VII

BIRTHRATE IN STANDARD POPULATION, AS COMPUTED BY SEVERAL METHODS

Method (a)	Direct solution of equation (2), arithmetical integration	.02088
" (b)	Same, integration by planimeter	.02092
" (c)	Solution of equation (41)	.02091
" (d)	From age distribution and maternity frequency	.02089
	Mean	.02090

Relation of Exact Value of b to Approximation. The relation of the exact value of b to the approximate value given in the text becomes apparent if we multiply together equations (2) and (3). We thus find, after expanding the exponentials by Taylor's theorem, and using the notation of (13) and (40):

$$\frac{1}{b}=\frac{R_0-rR_1+\ \ldots}{L_0-rL_1+\ \ldots} \tag{44}$$

If we neglect the terms containing r this gives

$$\frac{1}{b}=\frac{R_0}{L_0} \tag{45}$$

the formula given in the text, and there developed by an argument that requires the corroboration of the rigorous derivation here given.

Ratio of Males to Females Living. Some of the figures appearing in Table III involve the ratio of males to females in the standard population. It, therefore, becomes necessary to compute this, as follows:

The ratio of males N_m to females N_f in the living population differs from the sex ratio at birth. In a population of the type here considered the former ratio is given by[1]

$$\frac{N_m}{N_f}=\frac{B_m}{B_f}\frac{L_m+rL'_m}{L_f+rL'_f} \tag{46}$$

Here B_m and B_f denote total births per annum of males and females respectively. L denotes the mean length of life and L' the first moment of the life curve. The subscript indicates the sex. The rate of increase is r, and must, of course, ultimately be the same for both sexes. It is assumed that $m(a)$ is not very materially affected by moderate changes in the sex ratio.

Applying the formula (17) we find for the ratio of males to females in the standard population

$$\frac{N_m}{N_f}=1.017;\ \frac{N_m+N_f}{N_f}=2.017 \tag{47}$$

Birthrate of Both Sexes Combined. Having found the number of daughters per female, namely .02090, we can now find the birthrate (children of both sexes per population of both sexes). In 1920 the sex ratio at birth was 1.060. We then have

$$b=\frac{.02090\times 2.060}{2.017}=.02135.$$

This is the relation used in the computations appearing at the bottom of Table III.

METHOD OF USING LIFE TABLES IN THE RETROSPECTIVE COMPUTATION OF U. S. BIRTHRATES

The period covered (1872–1920) was divided into five ranges, as shown in the following table:

Period	Life table used as basis
1870–1885.....	Estimated,* 1880
1885–1895.....	Estimated,* 1890
1895–1905.....	Glover, 1901
1905–1915.....	Glover, 1910
1915–1920.....	Foudray, 1919–20

* As explained below.

[1] J. Washington Academy Science, 1913, p. 289.

and the computations were conducted on the basis that each life table was valid for a period of ten years centered at the year of the life table (except that the 1901 life table was for convenience centered at 1900).

The method of computation is best explained by the aid of a concrete example.

To find the birthrate in 1872, given the age distribution of the population in 1920:

1. Persons comprised within the age group 45 to 49 were born in the years 1870-1875. The central point of the age group 45–49 is age 47.5, and persons of this age in 1920 were born in 1872.

They lived from 1872–1885 under the régime of the 1880 life table, until they were 12.5 years old. Hence, if we denote by $l^{1880}_{12.5}$ the survival factor for 12.5 years of age according to the 1880 life table, 1,000 persons born would leave a surviving residue of $1{,}000\, l^{1880}_{12.5}$ to the year 1885.

These survivors then lived for 10 years under the régime of the 1890 life table. With a notation similar to that employed above, there would survive at the end of these 10 years, out of the original 1,000, a number equal to

$$1000\, l^{1880}_{12.5}\ \frac{l^{1890}_{22.5}}{l^{1890}_{12.5}}$$

Again, there will survive to 1900, a number

$$1000\, l^{1880}_{12.5}\ \frac{l^{1890}_{22.5}}{l^{1890}_{12.5}}\ \frac{l^{1900}_{32.5}}{l^{1900}_{22.5}}$$

Finally, by an obvious extension of this process, it is found that, of the original 1,000 persons born in 1872, the following number survive to 1920:

$$1000\, l^{1880}_{12.5}\ \frac{l^{1890}_{22.5}}{l^{1890}_{12.5}}\ \frac{l^{1900}_{32.5}}{l^{1900}_{22.5}}\ \frac{l^{1910}_{42.5}}{l^{1910}_{32.5}}\ \frac{l^{1920}_{47.5}}{l^{1920}_{42.5}}$$

$$= 1000\, l^{\circ}_{47.5}, \text{ say}$$

Now the number of native white females comprised within the age group 45–49 in 1920 was 1,853,613. Ascribing one-fifth of this number to the central age, 47 to 48, we have 370,723 females of average age 47.5 in 1920. These, then, are survivors of $\frac{370{,}723}{l^{\circ}_{47.5}} = \frac{370{,}723}{.60356} = 614226$ births in 1872.

Now the total white population in 1870 was 35,689,730 persons (of both sexes). Hence the birthrate of females was $\frac{614{,}226}{35{,}689{,}730} = .0172$.

Similarly, the birthrate of males was found to be .0201; thus the combined birthrate for the two sexes was .0373.

It may be remarked that this computation seems to give an excessively high sex ratio (males/females) at birth for the year 1872. But the example chosen is the most remote in time of the whole series; and the combined birthrate is probably a good deal nearer the truth than that of either sex separately. As will be seen by comparison with the English figures, the entire series has every appearance of versimilitude.

ESTIMATED POINTS IN LIFE TABLES

Two kinds of interpolations and exterpolations had to be made from the tabular figures available.

(a) For midyear points the arithmetic mean of l_x and l_{x+1} was taken.

(b) In the case of the Foudray life table l_x is given and at ages 2, 7, 12, etc., the values for 3, 8, 13, etc., are, however, readily obtained by the relation

$$l_{x+1} = p_x l_x$$
$$p_x = 1 - q_x$$

the value of q_x at ages 2, 7, 12, etc., being also given in the Foudray table.

ESTIMATED OLDER LIFE TABLES

Back beyond 1900 no U. S. life tables are available. An estimate of the required values of l_x for 1890 and 1880 was made by ascertaining the ratio $\frac{l_x^{1900}}{l_x^{1910}}$ at the particular age, and multiplying the 1900 l_x by this factor to obtain an estimated l_x for 1890. By a second similar multiplication l_x for 1880 was obtained.

A GENERAL MATHEMATICAL THEORY OF DEPRECIATION[1]

By Harold Hotelling, *Food Research Institute, Stanford University*

In the older treatments of depreciation the cost, or "theoretical selling price" of the product of a machine, was conceived of as determined causally by the addition of a number of items of which depreciation is one. In other words, depreciation was first computed by some rather arbitrary formula not involving the theoretical selling price, which was then found by the addition of depreciation to operating costs and division by quantity of output. It will be shown in this paper that depreciation and theoretical selling price must be computed simultaneously from a pair of equations which are frequently a bit complicated. The differences in the results obtained from the arbitrary and mathematical formulæ are often very large.

The simple methods referred to, which still prevail generally in business, are analogous to the naïve type of economic thought for which the only determiner of price is cost and which fails to consider the equally important rôle played by demand.

The "unit cost theory" gave the first recognition to the reciprocal relation existing between the value of a machine and the value of its product. It also deserves the credit for taking into account operating costs and output which, as in the present paper, are assumed to have been determined from experience. However, this theory in its usual form involves a number of serious errors of reasoning which have been pointed out by Dr. J. S. Taylor.[2]

As an improvement on the unit cost theory Dr. Taylor puts forward a method which, it is fair to say, is the only one that has been proposed which ever gives correct results. To it the present paper owes much. He assumes that unit cost plus (defined as operating cost, plus depreciation, plus interest on the value of the machine, all divided by the number of units of output) is to be determined by the conditions that

(a) it is either to remain constant during the machine's life or is to increase in a specified manner; and

(b) it is to be made a minimum.

This procedure is open to a possible criticism which he does not anticipate, namely, that, since the unit cost plus depends on the distribution of depreciation charges, there is danger of the accountant

[1] Presented before the American Mathematical Society (Chicago meeting), December 26, 1924.

[2] "A Statistical Theory of Depreciation Based on Unit Cost," this Journal, December, 1923, pp. 1010–1023.

deceiving himself into thinking that he has reduced costs when he has merely changed his system of charging depreciation to make the unit cost appear less. If unit cost is to be made a minimum in any useful sense, it is to be done by performing some operation on the machine itself, rather than by adjusting depreciation charges, or by any other process of bookkeeping. The criticism may be answered on Dr. Taylor's behalf by the fact that he uses the test (b) above only to determine the time for the tangible operation of scrapping the machine, and then uses (a) to distribute the depreciation charge over the useful life.

But even if we admit the validity of (a) there remains a question concerning (b). Does the manufacturer desire to make his unit cost plus, in the sense defined, a minimum? Or may not considerations of profit lead him to scrap the machine at some different time from that which makes unit cost plus a minimum, or to shut down his factory or restrict the output of the machine? We shall indeed give a proof that in certain cases the tests (a) and (b) give correct results.

To one important element in the problem I find no allusion in the literature. The value of an old machine or other property must depend upon the operating cost. In all depreciation theories based on unit cost it is assumed that the operating cost is known, and the value is then calculated. But operating cost always includes elements which depend on the value. Hence we cannot know the operating cost until we know the value; that is, until the problem is solved. We shall resolve this difficulty by solving a simple integral equation.

The viewpoint of the present treatment is that the owner wishes to maximize the present value of the output minus the operating costs of the machine or other property. This quantity is, in fact, the value of the property. The first step is, therefore, to set up an expression for the value of the property in terms of value of output, operating costs, and the life of the property. Various hypotheses concerning the economic situation are then applied to evaluate the unknowns appearing in the equation. Depreciation is defined simply as rate of decrease of value. A mathematical formulation of the problem of depreciation is presented from which, it is believed, the whole of the valid portions of existing depreciation theories are readily deducible, and which also lays a foundation for further developments. This treatment is adapted to the consideration of depreciation in connection with variation of such factors as prices and interest rates. The theory is in fact completely general. The property involved will be referred to as a machine purely for convenience of language. The amount of computation involved in a particular application will depend upon the degree of refinement

of the data; in the present stage of knowledge of mortality tables of property the work can be made quite simple. A specimen problem is worked out numerically.

Continuous functions, instantaneously compounded interest, and integration will be used as better adapted to the needs of theory than the discontinuous functions and summation processes usually employed. The methods of the integral calculus also appear to me simpler than those of algebraic summation even where only integral numbers of years are involved; while the occurrence of portions of years, which does not seriously affect the integration process, is always a source of trouble when discontinuous functions are used.

Obsolescence is a risk of essentially the same nature as fire, earthquake, or burglary, and should be provided for in the same manner—namely, by an allowance out of operating expenses similar to an insurance premium. Even where no insurance or inadequate insurance is carried, an allowance should be made. Obsolescence is thus to be relegated, with insurance, to the category of operating expenses.

FUNDAMENTAL FORMULA

If a machine with operating cost per year $O(\tau)$ produces $Y(\tau)$ units of output per year at time τ and if the value ("theoretical selling price") of a unit of output is x, the annual rental value of the machine at time τ is

$$R(\tau) = x \cdot Y(\tau) - O(\tau).$$

Now the value of a machine is the sum of the anticipated rentals which it will yield, each multiplied by a *discount factor* to allow for interest, plus the scrap or salvage value, also discounted.[1] In the

[1] The "force of interest" $\delta(t)$ is defined as the rate of increase of an invested sum s divided by s: $\delta(t) = \frac{1}{s}\frac{ds}{dt}$.

It follows by integration that $s = s_0 e^{\int_0^t \delta(\nu)d\nu}$, where s_0 is the value of the invested amount when $t = 0$ and ν is a mere variable of integration. Hence $s_0 = se^{-\int_0^t \delta(\nu)d\nu}$. That is, the present value s_0 of a payment to be made t years hence is the amount s of the expected payment multiplied by the *discount factor* $e^{-\int_0^t \delta(\nu)d\nu}$. If $\delta(\nu) = \delta$, a constant, the discount factor is $e^{-t\delta}$. The discount factor takes the form v^t if we write $v = e^{-\delta}$; and $v = 1/(1+i)$ in this case, where i is the *rate* of interest in the ordinary sense, *i.e.*, the interest payable at the end of a year on each dollar invested for the year. Then

$$\delta = \log_e(1+i) = i - \frac{i^2}{2} + \frac{i^3}{3} - \ldots$$

and

$$i = \delta - 1 = \delta + \frac{\delta^2}{2!} + \frac{\delta^3}{3!} + \ldots.$$

Since both these series converge rapidly they afford a ready means of passing from force of interest to rate of interest or vice versa. The difference between i and δ is ordinarily very small.

The value at time t of a payment to be made at time τ is evidently the amount of the payment multiplied by $e^{-\int_t^\tau \delta(\nu)d\nu}$. If s is constant, this discount factor equals $v^{\tau - t}$.

most general case the rate of interest will vary with the time. If $S(n)$ is a function giving the scrap or salvage value at the time n at which the machine is to be discarded, the value at time t is given by the following fundamental formula:

$$V(t)=\int_t^n [xY(\tau)-O(\tau)]e^{-\int_t^\tau \delta(\nu)d\nu}d\tau+S(n)e^{-\int_t^n \delta(\nu)d\nu}, \tag{1}$$

τ and ν being variables of integration representing time. In case the interest rate is constant (1) becomes

$$V(t)=\int_t^n [xY(\tau)-O(\tau)]v^{\tau-t}d\tau+S(n)v^{n-t} \tag{1a}$$

Since the value of a new machine is its cost c, we have for $t=0$,

$$c=V(0)=\int_0^n [xY(\tau)-O(\tau)]e^{-\int_0^\tau \delta(\nu)d\nu}d\tau+S(n)e^{-\int_0^n \delta(\nu)d\nu}. \tag{2}$$

Having once bought the machine, the owner wishes to conserve its value as far as he can. This, in the light of (1), is the same as saying that he hopes to get as much net rental as possible out of the machine, interest and scrap value being considered. We, therefore, adopt as the basis of our further work the simple postulate:

I. *Everything in the owner's power will be done to make $V(t)$ a maximum when $t>O$.*

A second postulate of less general validity, which is often assumed tacitly or is thought to follow from I, is the following:

II. *The machine is always operated at full capacity.*

We shall assume II tentatively.

We suppose that c and the functions Y, O, S and δ of the time are known; but before we can evaluate the right member of (1) we must make some hypothesis which will enable us to determine n and x. There are several possibilities.

(A) If the property has a known useful life n, (2) is a simple equation in x alone.

(B) If x is known, the most obvious way to find a value for n is to solve (2). This, however, can be justified only in very special circumstances. For if x is known in advance of the solution of our problem of depreciation, it must be determined by competitive conditions entirely beyond the control of the machine's owner. He may, for example, use the machine to do work usually done by some other process. In such a case (2) does not hold unless ideally fluid conditions of competition have acted to bring the price c of the machine to such a point as to justify its purchase when n is determined to best advan-

tage. Thus to use (2) to find the best life to allow the machine, the owner must rest precariously upon the judgment of his competitors.

The correct procedure is as follows. Let m be the time, if it exists (if not, put ∞ for m), at which the output of the machine ceases to be worth the operating expenses. Then m is determined by the equation $xY(m)-O(m)=0$. We may for the sake of generality consider that x is a given function $x(t)$ of the time. In this case m is determined by $x(m)\cdot Y(m)-O(m)=0$, which we may also write $R(m)=0$. The sum of the possible future rentals of the machine, discounted at (say) a constant rate of interest is then $\int_t^m R(\tau)v^{\tau-t}d\tau$. This quantity will decrease until, at time n, it reaches the scrap value $S(n)$. Thus n is determined by $\int_n^m R(\tau)v^{\tau-t}d\tau=S(n)$, where $R(m)=0$. It is of course possible that these equations may have a plurality of solutions. The correct solution is then the one which makes $V(t)$ greatest.

(C) If, as is most common, both x and n are to be determined, we use the equation (2) connecting these unknowns, solving it simultaneously with an additional equation obtained by means of I.

If all economic conditions are static, x is a constant. If economic conditions are not static but their trend can be estimated, we may consider that the value of a unit of output varies in a specified manner with the time, but that its general level depends upon an unknown parameter α. For example it may be supposed that x increases in geometric progression with known ratio r, but that the initial and final values are unknown. We then write $x=\alpha r^{\tau}$, or in general, $x=x(\tau,\alpha)$.

Take first the static case. Supposing all our functions continuous and with continuous first derivatives, we differentiate (1) and put $dV(t)/dn=0$. The result of this operation becomes, after cancelling out the common factor $e^{-\int_t^n\delta(\nu)d\nu}$ and solving [1] for x,

[1] The derivation of (3) given in the text is strictly accurate only in the case in which x is independent of n. If, as is generally the case, x is a function of n, the result of equating to zero the derivative of the right member of (1) with regard to n is

$$\frac{dx}{dn}\int_t^n Y(\tau)e^{-\int_t^\tau\delta(\nu)d\nu}d\tau+[xY(n)-O(n)-\delta(n)S(n)+S'(n)]e^{-\int_t^n\delta(\nu)d\nu}=0.$$

To get rid of dx/dn we differentiate also the relation (2) between x and n, obtaining an equation exactly like the one above excepting that t is replaced throughout by 0. Eliminating dx/dn between the two we have

$$\begin{vmatrix}\int_t^n Y(\tau)e^{-\int_t^\tau\delta(\nu)d\nu}d\tau & e^{-\int_t^n\delta(\nu)d\nu}\\ \int_0^n Y(\tau)e^{-\int_0^\tau\delta(\nu)d\nu}d\tau & e^{-\int_0^n\delta(\nu)d\nu}\end{vmatrix}[xY(n)-O(n)-\delta(n)S(n)+S'(n)]=0.$$

If the factor in the square brackets vanishes we have (3). But it must vanish, for the determinant becomes, when the upper row is multiplied by $e^{-\int_0^t\delta(\nu)d\nu}$,

$$x = \frac{O(n) + \delta(n)S(n) - S'(n)}{Y(n)}, \tag{3}$$

a condition independent of t, as it should be. This equation states that x, the cost of a unit of product is found by adding the operating cost $O(n)$ of the machine (at the time n when it is least efficient and is about to be scrapped) to interest $\delta(n)S(n)$ on the scrap value and the rate of depreciation $-S'(n)$ of the scrap value, and dividing this sum by the machine's rate of production.

If conditions are not static we may still be able to write a known function $x(\tau,\alpha)$ for x in (1) and (2), and $x(n,\alpha)$ for x in (3). We then solve (2) and (3) simultaneously for n and α and substitute in (1). This parallels a method suggested by Dr. Taylor in the paper cited above, but is slightly more general.

There is of course nothing in the theory to preclude the possibility that n may become infinite.

OPERATING EXPENSE DEPENDENT ON VALUE

It has been assumed up to this point that when we set out to solve a particular depreciation problem we know a function $O(t)$ giving the operating expense at all times in the life of the article. But certain elements of operating cost always depend upon the value of the article at the time, which value we do not know until we have solved the problem. Thus taxes are supposed to be proportional to the value, and insurable risk, if not insurance, certainly is. The risk of obsolescence enters here.

If we write $O(\tau)$ in (1) as a function of $V(\tau)$ and τ we have an *integral equation* to solve for the unknown function $V(t)$. Now the study of integral equations is a new and incomplete branch of mathematics, so that an integral equation written down at random can probably not be solved until pure mathematics has advanced further than at present. By rare good luck, however, the cases which usually arise in practice are of a type leading to one of the few integral equations whose theories are well developed, the *Volterra* equation. This is because the dependence of operating cost upon value is ordinarily *linear*. Thus

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$$\begin{vmatrix} \int_t^n Y(\tau)e^{-\int_0^\tau \delta(\nu)d\nu}d\tau & e^{-\int_0^n \delta(\nu)d\nu} \\ \int_0^n Y(\tau)e^{-\int_o^\tau \delta(\nu)d\nu}d\tau & e^{-\int_o^n \delta(\nu)d\nu} \end{vmatrix},$$

which can vanish only if the terms in the first column are equal. This would imply that $\int_0^t Y(\tau)e^{-\int_0^\tau \delta(\nu)d\nu}d\tau = 0$, which is impossible (excepting for $t=0$) because we suppose $Y(\tau)$ positive for all values of τ less than n.

taxes and insurance premiums are directly proportional to the value, and not to its square or some other function. Hence we may write

$$O(\tau) = A(\tau) + B(\tau)V(\tau),$$

where $A(\tau)$ and $B(\tau)$ are functions which, like $Y(\tau)$ and $\delta(\tau)$, are supposed to have been determined, or at least estimated, on the basis of experience. Substituting this value of $O(\tau)$ in (1) we have

$$\left.\begin{aligned} V(t) = &\int_t^n [x \cdot Y(\tau) - A(\tau)]e^{-\int_t^\tau \delta(\nu)d\nu}\delta\tau + S(n)e^{-\int_t^n \delta(\nu)d\nu} \\ &- \int_t^n B(\tau)V(\tau)e^{-\int_t^\tau \delta(\nu)d\nu}d\tau. \end{aligned}\right\} \tag{4}$$

This integral equation admits of a very easy solution by reduction to a differential equation as follows. Differentiate (4) and add each member of the resulting equation to the product of $-\delta(t)$ by the corresponding member of (4). Several terms cancel out, leaving [1]

$$\frac{dV(t)}{dt} - [\delta(t) + B(t)]V(t) = -xY(t) + A(t). \tag{5}$$

Let us write for brevity $\gamma(t) = \delta(t) + B(t)$. The solution of (5), found by elementary methods, is then

$$V(t) = e^{\int_0^t \gamma(\nu)d\nu}\left\{\int_t^n [xY(\tau) - A(\tau)]e^{-\int_0^\tau \gamma(\nu)d\nu}d\tau + k\right\}$$

where k is the constant of integration. To evaluate k we may use either the fact that $V(0) = c$ or that $V(n) = S(n)$. In the first case we find

$$V(t) = e^{\int_0^t \gamma(\nu)d\nu}\left\{c - \int_0^t [xY(\tau) - A(\tau)]e^{-\int_0^\tau \gamma(\nu)d\nu}d\tau\right\};$$

in the second case we obtain the equivalent form

$$V(t) = \int_t^n [xY(\tau) - A(\tau)]e^{-\int_t^\tau \gamma(\nu)d\nu}d\tau + S(n)e^{-\int_t^n \gamma(\nu)\delta\nu}, \tag{1a}$$

which differs from (1) only in that $\gamma(\nu)$ replaces $\delta(\nu)$ and $A(\tau)$ replaces $O(\tau)$.

The problem of value and depreciation is now solved exactly as in the simpler case. Equations similar to (2) and (3) are derived and used in the same manner as before, δ being everywhere replaced by γ and $O(t)$ by $A(t)$.

[1] If we solve (5) for x we have the well-known expression for cost of output in terms of operating costs, yield, interest and depreciation on the machine.

EXAMPLE

Suppose that the data available from experience justify the expectation that

$$Y(\tau)=ae^{-\lambda\tau},\ A(\tau)=be^{\mu\tau},\ S(n)=0, \text{ and } \delta(\nu)+B(\nu)=\gamma, \text{ a constant.}$$

Then, by (1a),

$$\begin{aligned}V(t)&=\int_t^n[xae^{-\lambda\tau}-be^{\mu\tau}]e^{-\gamma(\tau-t)}d\tau\\&=e^{\gamma t}\int_t^n[xae^{-(\lambda+\gamma)\tau}-be^{(\mu-\gamma)\tau}]d\tau\\&=xa\frac{e^{-\lambda t}-e^{-(\lambda+\gamma)n}}{\lambda+\gamma}+b\frac{e^{\mu t}-e^{\gamma t+(\mu-\gamma)n}}{\mu-\gamma}.\end{aligned}$$

From an equation similar to (3) we have, since $S(n)=0$ and therefore $S'(n)=0$, $x=A(n)/Y(n)=be^{(\lambda+\mu)n}/a$.

Substituting above we have

$$V(t)=\frac{b}{(\lambda+\gamma)(\mu-\gamma)}\left\{(\mu-\gamma)e^{-\lambda t+(\lambda+\mu)n}-(\lambda+\mu)e^{\gamma t+(\mu-\gamma)n}+(\lambda+\gamma)e^{\mu t}\right\}.$$

For $t=0$ this gives

$$c=V(0)=\frac{b}{(\lambda+\gamma)(\mu-\gamma)}\left\{(\mu-\gamma)e^{(\lambda+\mu)n}-(\lambda+\mu)e^{(\mu-\gamma)n}+\lambda+\gamma\right\},$$

which on rearrangement becomes

$$e^{(\lambda+\mu)n}+\frac{\lambda+\mu}{\gamma-\mu}e^{(\mu-\gamma)n}+\frac{\lambda+\gamma}{\mu-\gamma}-(\lambda+\gamma)\frac{c}{b}=0.$$

This equation for determining n, it will be observed, contains b and c only in the ratio c/b, and does not contain a at all. Since commensurable values may always be taken for γ, λ and μ it reduces to an algebraic equation. It may be proved mathematically that in every case there is just one root for which $n>0$, and that n approaches zero and infinity with c/b.

Let us take $\gamma=.09$, $\lambda=.06$, $\mu=.14$, $c/b=20$. The equation becomes $e^{.20n}-4e^{.05n}=0$. Hence $e^{.05n}=\sqrt[3]{4}$, and $n=\frac{1}{3}\log 4/.05\log e=9.244$ years. Therefore

$$\begin{aligned}V(t)&=\frac{b}{.15}[e^{-.06t+.20n}-4e^{.09t+.05n}+3e^{.14t}]\\&=\frac{b}{.15}[4^{4/3}(e^{-.06t}-e^{.09t})+3e^{.14t}].\end{aligned}$$

Calculation from this formula gives the following results for $b = 1{,}000$.

t	$V(t)$	$V(t-1)-V(t)$
0	20,000.00	
1	16,553.86	3,446.14
2	13,327.50	3,226.36
3	10,345.07	2,982.43
4	7,638.10	2,706.97
5	5,246.63	2,391.57
6	3,220.65	2,025.98
7	1,621.75	1,598.90
8	524.96	1,096.79
9	20.97	503.99
9.244	0	

The decrease in value shown by this table for the first year is more than $1\frac{1}{2}$ times that indicated by the straight line law, and is more than double that calculated by means of the equal annual payment or sinking fund method with interest at 6 per cent.

Depreciation has been defined as rate of decrease of value: $D(t) = -dV(t)/dt$.

The *total depreciation* over a period is the difference between the value at the beginning of the period and that at its end. It equals the average value of the depreciation times the length of the period; for

$$\int_a^b D(t)dt = -\int_a^b \frac{dV(t)}{dt}dt = V(a) - V(b).$$

The total depreciation in the value of an article over its whole life, is thus $c - S(n)$. The average depreciation is therefore

$$\frac{c-S(n)}{n}. \tag{6}$$

RELATION TO OTHER DEPRECIATION THEORIES

The formula (6) for average depreciation is so simple that if accuracy is not worth while, and if the useful life n is known with some degree of definiteness, it may be assumed that the depreciation of each year is equal to this average depreciation. Or if we are considering the composite depreciation of a large number of similar machines whose times of installation have been uniformly distributed over n years, (6) gives a rough approximation. This *straight line law* is often used in cases in which it gives rise to large errors.

Let us suppose that $O(t)$ and $Y(t)$ are constants q and y, respectively, for a certain number of years of the machine's life and then change abruptly in such a way as to make it clear that the end has come. The supposition that $O(t)$ is constant means, of course, that $B(t)=0$, that is, that neither risk nor a tax on value exists. Let the scrap value be s, a constant, and let $\delta(t)=\delta$, a constant. Let x also be constant. Then (3) does not hold because its derivation assumed $O(t)$ and $Y(t)$ continuous. But n is now known, so that we can find x from (2) alone.

In this case (1) becomes

$$V(t)=\int_t^n (xy-q)v^{\tau-t}\,d\tau+sv^{n-t}$$
$$=(xy-q)\frac{1-v^{n-t}}{\delta}+sv^{n-t}.$$

Putting $t=0$ we have the equation, like (2),

$$c=(xy-q)\frac{1-v^{-n}}{\delta}+sv^{n}.$$

The result of eliminating $xy-q$ between these two equations is

$$\begin{vmatrix} V(t)-sv^{n-t} & \dfrac{1-v^{n-t}}{\delta} \\ c-sv^{n} & \dfrac{1-v^{n}}{\delta} \end{vmatrix}=0.$$

Now add $-s\delta$ times the second column to the first. Then subtract the second row from the first. Finally, multiplying the second column by $(1+i)^n$ and remembering that $\bar{s}_{\overline{n}|}=\dfrac{(1+i)^n-1}{\delta}$, we have

$$\begin{vmatrix} V(t)-c & -\bar{s}_{\overline{t}|} \\ c-s & -\bar{s}_{\overline{n}|} \end{vmatrix}=0.$$

From this we find at once $V(t)=c-(c-s)\dfrac{\bar{s}_{\overline{t}|}}{\bar{s}_{\overline{n}|}}=c-(c-s)\dfrac{s_{\overline{t}|}}{s_{\overline{n}|}}$.

Hence
$$V(t)-V(t+1)=(c-s)\frac{(1+i)^t}{s_{\overline{n}|}}. \tag{7}$$

This gives a proof of the correctness, under the circumstances assumed, of the "equal annual payment" and "sinking fund" methods of calculating depreciation. But these methods are almost always misleading because the assumptions take no account of the almost uni-

versal tendency for operating costs to increase and for output to decrease with age. Consequently they understate the depreciation in the early years.

We may further inquire in what circumstances particular depreciation methods are valid. The test here is supplied by (5), which we now write in the form

$$P(t) = \gamma V(t) - \frac{dV(t)}{dt}, \tag{8}$$

where $P(t) = xY(t) - A(t)$. Let us, for simplicity, consider γ constant, and let us call $P(t)$ the *gross rental.*

According to the straight line method,

$$V(t) = c - (c-s)t/n.$$

Substituting this expression in (8) we find

$$P(t) = c - s + c\gamma - (c-s)\gamma t/n.$$

This is such a special condition that we must in general discard the straight line method as a valid rule where accuracy is required.

The reducing balance method supposes that depreciation is a constant percentage of value. It follows that $V(t) = ce^{-kt}$, k being determined by $s = ce^{-kn}$. From (8) we have in this case the condition $(P)t = ce^{-kt}(\gamma - k)$.

Similarly the condition for the validity of the sinking fund method (7) is

$$P(t) = \gamma - \frac{c-s}{(1+i)^n - 1}[B(t)(1+i)^t + \gamma],$$

which is constant if $B(t) = 0$.

Thus the three methods in most common use depend for their validity upon the satisfaction of conditions which involve no disposable constants and which are so special that the chances are overwhelmingly against the satisfaction of any of them in a particular case.

Dr. Taylor [1] defines unit cost plus by a formula which, in terms of continuous functions, becomes

$$x = \frac{O(t) + \delta(t)V(t) - \dfrac{dV(t)}{dt}}{Y(t)},$$

which is equivalent to (5), and reduces to (3) when $t = n$. We may now derive his criterion of minimum unit cost plus as follows. The value of

[1] J. S. Taylor, *op. cit.*

the machine being given by (1) in terms of x and n, and these unknowns being connected by (2), we seek the value of n which will make x a minimum. This can be found from (2) alone. For, differentiating with respect to n and putting $dx/dn = 0$, we have (3). The simultaneous solution of (2) and (3) will then yield the same values of x and n as those found under (C).

Thus Dr. Taylor's treatment yields the same results as those found under (C), save for the discrepancies arising from the use of discontinuous functions which suppose all changes to occur by yearly steps. It is better adapted to calculation than the theory of the present paper in many cases in which $B(t) = 0$, that is, in which there is no element of operating expense such as risk, insurance and taxes. But even under this severe restriction it does not seem a logical foundation for a theory of depreciation without a demonstration, such as the one above, that the life to be allowed a machine to make it most valuable to its owner is also that which makes unit cost plus a minimum. This proposition is not obvious, and is false in case of the failure of our postulate II.

Dr. Taylor in the same paper mentions as an alternative to that discussed—though without advocating it—a theory based on the assumption that "unit cost," defined like unit cost plus except that interest is not included, should be made a minimum. This assumption lacks even the justification given for minimizing unit cost plus. Like the straight line law, it has nothing to recommend it but simplicity.

FURTHER DEVELOPMENTS

What can be said in case of the failure of postulate II—that the property is used to full capacity? There are very important cases, such as those of mines, in which the postulate is not even approximately true. In this connection we must consider as unknowns not only useful life, value, and depreciation but also the functions $Y(t)$ and $A(t)$. The owner, that is, may voluntarily run the machine at less than full capacity, and wishes us to tell him just how fast to let it run in order that his profits may be a maximum. If we do not tell him, he will guess to the best of his ability. The demand function must be known in order to give a solution.

In all such cases the guiding principle is that the right member of (1), representing discounted future profits, is to be made a maximum. Even if the capitalist system is to give way to one in which service and not profit shall be the object, there will still be an integral of anticipated utilities to be made a maximum. Since we must find a function which maximizes an integral we must in many cases use the Calculus of Variations. But the problem here transcends the questions of depre-

ciation and useful life, and belongs to the dawning economic theory based on considerations of maximum and minimum which bears to the older theories the relations which the Hamiltonian dynamics and the thermodynamics of entropy bear to their predecessors.[1]

The question of charging depreciation as a function of output rather than of time has been discussed of late.[2] It is more natural to consider the depreciation of an automobile in terms of miles than of years. Since interest is an element, this is strictly possible only in case the rate of output $Y(t)$ is known for the whole life of the property; but in this case the arguments commonly advanced for the proposed method fail.

However, an answer may be given to a question of some interest which arises in this connection in case operating cost depends on a controllable rate of output and not merely on the time. What is the additional cost of taking a short trip in an automobile? More generally, what is the additional net cost of a slight increment in output of a machine? We suppose that the increased output extends over a time so short that interest for this period is negligible. The operating cost $A(t)$ per unit of time, exclusive of items such as risk and taxes which are proportional to value, may usually be divided into three classes. One class of costs depends only on the age of the machine, one on age and the amount of current use, and one on the total of past use. We may thus write as an approximation which is probably good enough for all existing data,

$$A(t)=\alpha(t)+a(t)Y(t)+b\int_o^t Y(\mu)d\mu.$$

The additional cost of a small temporary increment z to the total output, concentrated in a time dt, consists partly of increased operating cost at this time and partly of depreciation due to subsequent increase in operating cost. The first part is obviously $za(t)$. The second part depends upon the increment of the integral $\int_o^\tau Y(\mu)d\mu$. This increment will be z for every time τ later than the time t at which the increased use takes place. Thus the future rentals of the machine are decreased by an amount bz per unit of time. The decrease in its value is the discounted sum of these decreases,

[1] For the solution of a special problem in the new "entropy" economics, see G. C. Evans, "The Dynamics of Monopoly," *American Mathematical Monthly* XXXI, 2, February 1924, pp. 77–83. A thorough working knowledge of the Calculus of Variations is a prerequisite to the development of this type of economic theory—which doubtless explains why it has not developed further.

All hedonistic and eudaemonistic ethical theories, which declare that the total of pleasure or happiness should be made a maximum, really reduce the question of right conduct to a set of problems in the Calculus of Variations and in the more general theory of maxima of functionals.

[2] E. A. Saliers, *Depreciation, Principles and Applications*, Ronald Press, 1922, pp. 172–178.

$$\int_t^n bze^{-\gamma(\tau-t)}d\tau = bz\frac{1-e^{-\gamma(n-t)}}{\gamma},$$

if γ is constant.

SUMMARY

(A) The value of a machine and that of a unit of its output are *interrelated*, each affecting the other. This economic truism must underlie a correct theory of depreciation.

(B) The fundamental formula (1) gives the value of a machine in terms of time, value of output, operating cost, scrap value, useful life and rate of interest. One relation between these quantities, given by (2), is due to the fact that the cost of the machine when new is its value at that time.

(C) Two postulates are introduced, one equivalent to the supposition of completely rational action with a completely selfish motive; the second, to which we attach less permanent importance, excludes from consideration the possibility that the owner may seek to increase his profits by slowing down production.

(D) Methods are given for finding the value function $V(t)$ when we know either the useful life n of the machine or the value x of a unit of output, and also when we know neither n nor x.

(E) The hitherto untouched difficulty of operating expenses proportional to value is resolved by means of an integral equation.

(F) A numerical example is worked out which is believed typical of a large class of cases.

(G) The depreciation methods hitherto used are tested and all excepting that of Dr. Taylor are found in general to give false results.

(H) When postulate II is abandoned and the rate of output considered controllable, the methods of this paper are capable of further elaboration to cover a great deal of economic and even ethical theory.

(I) Depreciation cannot, as has been proposed, be charged as a function of output alone, the omnipresence of interest preventing this. The increase of depreciation with output can however be calculated by means of our formulae, given adequate experience tables.

Since this article was written, two papers bearing on the subject have appeared: "Economics and the Calculus of Variations," by G. C. Evans, *Proceedings of the National Academy of Sciences*, Vol. 11, p. 90; and "A Note on the Theory of Depreciation," by J. S. Taylor, *Bulletin of the American Mathematical Society*, Vol. 31, p. 222.

A NEW ZEALAND STUDY IN INFANT MORTALITY

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Special interest attaches to a study of the infant mortality of the Dominion of New Zealand, by reason of the fact that these remote isles enjoy the reputation of possessing "unequivocally the lowest rate of infant mortality in the world".[1] There is abundant evidence moreover that the statistics are very accurate, and that the low rate of infant mortality can in no way be attributed to a coexistence of complete birth-registrations in the face of defective death-registrations.[2] (The figures quoted exclude the Maori population.)

Viewing the problem of infant mortality as part of the general problem of mortality, it may be interesting to consider the percentage which the deaths of children under one year of age have from time to time borne to total mortality, each sex being treated separately.

TABLE I

PERCENTAGE TO TOTAL MORTALITY AT ALL AGES OF DEATHS OF INFANTS UNDER 12 MONTHS, EACH YEAR, 1872 TO 1923

Year	Percentage		Year	Percentage	
	Males	Females		Males	Females
1872	33	36	1898	20	22
1873	32	35	1899	23	25
1874	32	36	1900	20	21
1875	31	34	1901	19	20
1876	32	37	1902	19	22
1877	30	36	1903	20	22
1878	30	35	1904	20	20
1879	33	38	1905	19	21
1880	31	36	1906	18	19
1881	30	33	1907	22	22
1882	28	32	1908	19	20
1883	31	35	1909	18	18
1884	26	29	1910	18	19
1885	27	31	1911	15	16
1886	29	33	1912	15	16
1887	27	32	1913	16	17
1888	23	24	1914	14	15
1889	24	27	1915	14	14
1890	22	26	1916	14	14
1891	25	27	1917	13	13
1892	24	26	1918*	..	..
1893	23	24	1919	10	10
1894	21	23	1920	13	12
1895	23	24	1921	13	13
1896	22	23	1922	12	10
1897	20	21	1923	11	10

* Abnormal on account of influenza epidemic.

[1] Edward B. Reuter, *Population Problems*, Lippincott, 1923, p. 156.

[2] E. P. Neale, "A New Zealand Study in Seasonal Fluctuations of External Migration," *Journal of the Royal Statistical Society*, March 1923, pp. 226 to 230.

The table shows a marked decline in infant mortality, but it must not be overlooked that part of this decline is fictitious. In the "seventies", New Zealand had only just been, and was still being, settled by immigrants in the prime of life, and the older age-groups were virtually devoid of occupants. Accordingly, the high proportion of infantile to total deaths in the earlier years covered by the table is in part due to the fact that at this time children formed an abnormally high proportion of the total population, the more so as at that time the actual fecundity of the population was much greater than it has been in recent years.

It will be observed that infant mortality almost invariably constituted a higher proportion of total mortality among females than among males. This difference again is mainly, if not entirely, fictitious, being the result of the fact that the middle and higher age-groups amongst males have been largely recruited from overseas as well as by natural increase, while amongst females the recruiting from overseas has been relatively less marked. This was especially so in the "seventies" and earlier. Since then New Zealand has become more settled, and therefore a more attractive country for women to enter from abroad. This fact, and the declining importance of net immigration relative to natural increase as factors in population growth, account for the progressive decline in the magnitude of the discrepancies between the figures for the two sexes.

An approach to the problem of infant mortality from the point of view of its relation to total mortality is thus seen to be an unsafe road by reason of the disturbing effects of variations in the age constitutions of the populations concerned upon the figures deduced. A more reliable index of infant mortality would be the proportion of deaths of children under one year to the total population under one year. In practice, however, owing to the difficulty of arriving at a figure for the total population under one year, it is usual to compute "rates of infantile mortality," the procedure being to divide the number of registrations of deaths of children under one year of age by the number of registrations during the same period of children born alive, the quotient being then multiplied by 1,000. Some of the deaths occurring this year really relate to births occurring last year, and a correction for this violation of the strict letter of the statistical axiom—that coefficients should represent ratios between logically agreeing magnitudes—should be applied in work requiring meticulous accuracy. In practice, however, the errors tend to compensate so long as there are no violent annual fluctuations in birth-rates. It is minimized moreover by the fact that the New Zealand law requires a much prompter registration

of deaths (from three to seven days are allowed, according to locality) than of births. Up to sixty-two days are allowed, and under special circumstances even longer.[1]

It has frequently been noticed that, broadly speaking, a high actual birth-rate and a high rate of infantile mortality go hand in hand, as do also a low birth-rate and a low rate of infantile mortality.[2] It is not difficult to adduce reasons why this should be so. Large families mean less time and diminished funds available for care and attention in sickness and in health of the individual members of those families.

Much that has been said and written on the subject of the birth-rate has involved the overlooking of the fact that the real importance attaches for most purposes not so much to the actual birth-rate as to what is sometimes known as the effective, sometimes as the residual, birth-rate. Such a birth-rate is computed, not on the basis of the number of children born, but on the number born and surviving for a given period (usually 12 months) after birth. It aims to show the rate at which children are born who have a reasonable chance of ultimately becoming effective members of society. To avoid vitiation of comparisons through changes in the age and sex compositions of populations, such rates are possibly best computed with the female population 15 and under 45 years, rather than the total population, as denominator. In this connection Table II is of interest.

TABLE II

BIRTHS, ACTUAL AND EFFECTIVE, PER 1,000 WOMEN 15 AND UNDER 45 YEARS, FROM 1876 TO 1923

Average of years	Actual births per 1,000 women 15 and under 45 years	Effective births per 1,000 women 15 and under 45 years
1876–1879	214	194
1880–1883	198	180
1884–1888	164	150
1889–1893	139	128
1894–1898	118	109
1899–1903	111	102
1904–1908	114	106
1909–1913	112	105
1914–1918	104	99
1919–1923	97	93

Effective birth-rates declined considerably during the period reviewed, but not so greatly as the actual birth-rates. This is because the infant mortality has declined in sympathy with the decline in

[1] Notification, as distinct from registration, of a birth is, however, required within 48 hours if the birth occurred in a city or borough, within 21 days if it occurred elsewhere.

[2] G. H. Knibbs, *Mathematical Theory of Population*, Australian Bureau of Census and Statistics, 1917, p. 149.

actual birth-rates, but the decline in the former case has been insufficient fully to counteract the decline in the latter case.

It follows from what has already been said that a given effective birth-rate may be consistent with either: (a) a high actual birth-rate and a high rate of infant mortality; or (b) a low actual birth-rate and a low rate of infant mortality. Between these two extremes all grades of transition are possible. Which of the two extremes is preferable, and why?

It is submitted that a high birth-rate, with its normal accompaniment of high infantile mortality, is undesirable on the following grounds:

1. Frequent pregnancies and confinements on the part of women mean much physical discomfort, mental anxiety, and expense for both parents.

2. Mothers, prospective and actual, are rendered temporarily unable to do full justice to their household and other duties.

3. Most parents with large families, are unable owing to the enormous demands on their time and incomes, to provide sufficient food, clothing, attention, etc., for their little ones, or indeed for themselves.

4. The consequent greater liability to sickness and want lead to frequent funerals, with their attendant expense, and to the filling of the cemeteries with tiny mounds. "Keep the cradles full," and the undertakers will never be out of a job.

5. Much economic waste is involved in the rearing to an early death of infants who, succumbing to the conditions of their environment, are destined never to become effective members of society.

The birth-rate influences the rate of infantile mortality, for reasons which we have already considered. Is the converse true: would we be justified in expecting a high rate of infant mortality (in connection, for example, with some epidemic carrying off infants in large numbers) to be associated with a high birth-rate? For physiological reasons, we might expect the death of an infant to subject the mother to the risk of maternity slightly sooner on the average than would have been the case had the infant survived, thus tending to increase the birth-rate. But it is an admitted fact that under modern conditions prudential as well as physiological considerations exercise an influence on fecundity that cannot be ignored. And it would appear that here for once prudential and physiological factors reinforce each other. For under modern conditions it is reasonable to suppose that the tendency will be for parents losing an infant by death to desire deliberately to replace it. The desire for offspring is probably sufficient up to a certain point to counteract neo-Malthusian influences, so that where a child is lost to a family which, but for its loss, would have suffered no further addi-

tions to its numbers, the tendency will be, at least in many cases, for the place of that dead child to be filled as a result of deliberate intention on the parents' part. From Table III it will be seen that 1899, 1903, and 1907 were all years of a high infant mortality from epidemic diseases, and in every case were followed by otherwise unexplainable rises in the birth-rate. These are the only years of really high epidemic infant mortality since the New Zealand birth-rate attained comparative stability at a fairly low level, that is, since prudential considerations have come to hold paramount sway.

Table III and the accompanying chart clearly bring out the secular decline in the rates of infant mortality. The recent great improvement dates from the first decade of the new century, and its commencement synchronizes roughly with the introduction of compulsory registration of midwives (1904) and the inauguration (at first locally, in Dunedin, in 1907) of the activities of the Royal New Zealand Society for the Health of Women and Children, commonly known as the Plunket Society. The Midwives Act provided that no women not in practice in 1904 (and even then only persons with three years' experience and of good moral character) may practise as midwives unless they have gone through the prescribed course of training and qualified for a certificate. On March 31, 1923, there were 1786 registered midwives, of whom 1541 were trained and certificated, the remainder having satisfied the Registrar that they complied with the alternative provisions of the Act. The Plunket Society, which is enabled to carry on by voluntary subscriptions supplemented by State subsidies, has published under the editorship of Dr. F. Truby King, pamphlets "which reduce to simple language the teachings of science and common sense as to the rearing of infants." One of these pamphlets is now issued gratuitously at the expense of the New Zealand Government to all married women of childbearing age in the Dominion. The Society has also established "Karitane Hospitals" for the care of weakly infants, while the services of its "Plunket nurses" are available gratuitously for advising mothers. Much good work has also been done by the antenatal clinics and baby clinics established in connection with the Government Maternity Hospitals in seven of the principal centers of the Dominion. The first of these hospitals was opened in 1905. During the year ended March 31, 1923, a total of 1531 confinements (between 5 and 6 per cent of the total confinements in New Zealand) took place in these institutions, while an additional 570 confinements were attended by the institution nurses outside these hospitals. They are primarily institutions for providing better care in confinements for the wives of working men. Some of the recent declines in the rates of

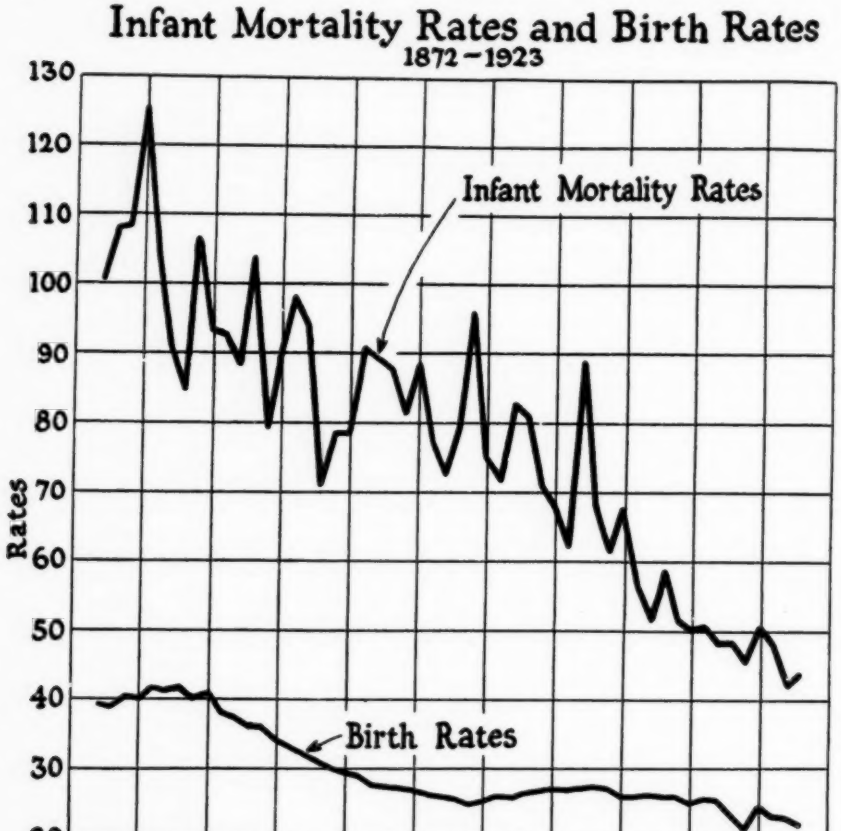
TABLE III

INFANT MORTALITY RATES FROM VARIOUS CAUSES, AND BIRTH-RATES, EACH YEAR, 1872 TO 1923

Year	Birth-Rates	Infant Mortality Rates from Various Causes								
		Epidemic Diseases	Tuberculosis	Convulsions	Respiratory Diseases	Gastric and Intestinal Diseases	Malformations	Early Infancy	Other Causes	Totals
1872.....	39.5	12.8	5.1	10.7	10.8	24.3	0.6	24.5	11.6	100.4
1873.....	39.0	26.8	4.2	9.9	12.7	15.7	0.8	22.8	15.2	108.1
1874.....	40.3	10.5	4.8	9.5	13.9	28.9	0.5	23.7	16.7	108.5
1875.....	40.2	8.6	7.2	10.7	17.4	26.4	1.4	29.3	24.8	125.8
1876.....	41.7	8.6	5.9	7.7	9.9	25.7	2.6	24.8	18.3	103.5
1877.....	41.3	10.8	5.5	7.2	10.1	18.8	1.5	21.3	15.4	90.6
1878.....	41.8	9.8	4.7	7.4	8.9	18.3	0.9	21.0	13.4	84.4
1879.....	40.3	13.8	5.4	8.0	15.9	22.9	1.3	24.5	15.6	107.4
1880.....	40.8	6.6	5.0	7.7	13.6	19.9	1.6	21.4	17.5	93.3
1881.....	38.0	10.1	5.4	7.5	13.1	19.0	1.4	21.2	14.7	92.4
1882.....	37.3	8.7	4.9	8.3	11.4	15.3	1.4	24.9	13.4	88.3
1883.....	36.3	12.6	5.2	8.3	12.8	24.8	1.0	22.7	16.5	103.9
1884.....	35.9	8.0	4.7	8.0	10.1	11.4	1.4	25.3	10.4	79.3
1885.....	34.3	7.3	3.9	6.8	11.9	19.8	0.9	27.8	10.8	89.2
1886.....	33.1	9.9	4.9	8.2	12.5	24.2	1.2	27.0	10.5	98.4
1887.....	32.1	9.9	5.3	6.6	9.9	24.4	1.4	25.5	10.8	93.8
1888.....	31.2	6.6	4.2	6.1	9.4	11.9	1.3	22.3	8.9	70.7
1889.....	30.1	4.6	2.6	5.7	10.4	19.3	1.3	25.9	9.1	78.9
1890.....	29.4	8.6	2.6	6.2	9.5	18.6	1.4	24.7	7.1	78.7
1891.....	29.0	14.7	3.6	6.7	13.4	18.4	1.1	25.0	8.3	91.2
1892.....	27.8	8.8	3.6	6.5	10.6	21.6	0.9	27.2	10.0	89.2
1893.....	27.5	13.7	3.3	6.7	14.3	12.9	1.6	25.8	9.7	88.0
1894.....	27.3	11.2	3.2	5.7	8.9	13.4	1.0	22.5	15.4	81.3
1895.....	26.8	10.0	3.5	7.3	15.0	15.4	2.0	24.4	10.7	88.3
1896.....	26.3	5.4	2.8	6.8	6.2	19.7	1.6	24.8	10.0	77.3
1897.....	26.0	4.0	2.7	5.8	9.7	17.1	1.6	23.0	8.4	72.3
1898.....	25.7	5.9	2.7	5.3	7.5	19.5	1.5	28.2	9.1	79.7
1899.....	25.1	9.7	3.5	6.8	13.8	20.6	1.3	29.9	10.3	95.9
1900.....	25.6	6.6	1.8	5.4	8.0	16.4	1.5	22.7	12.8	75.2
1901.....	26.3	4.2	2.3	4.4	11.0	12.5	1.7	27.3	8.0	71.4
1902.....	25.9	7.1	1.8	4.0	12.2	17.3	1.6	31.2	7.7	82.9
1903.....	26.6	10.4	2.1	4.7	11.5	15.4	1.6	27.7	7.7	81.1
1904.....	26.9	4.0	1.2	4.2	7.2	18.2	1.4	26.2	8.6	71.0
1905.....	27.2	3.0	1.2	4.1	10.0	14.2	0.9	27.0	7.1	67.5
1906.....	27.1	3.0	1.3	3.7	7.5	11.5	1.1	25.8	8.2	62.1
1907.....	27.3	12.5	1.9	3.3	11.4	22.8	1.3	27.7	7.9	88.8
1908.....	27.5	3.7	1.5	2.9	6.0	17.8	1.6	27.4	7.0	67.9
1909.....	27.3	3.7	1.2	4.0	7.2	11.3	2.2	25.2	6.8	61.6
1910.....	26.2	6.6	1.4	3.1	6.9	15.0	2.2	27.6	4.9	67.7
1911.....	26.0	2.8	0.6	3.4	6.6	10.7	1.9	25.4	4.9	56.3
1912.....	26.5	2.1	1.1	1.9	5.1	7.9	4.3	26.8	2.0	51.2
1913.....	26.1	4.1	0.7	2.5	6.2	9.0	3.2	28.4	5.1	59.2
1914.....	26.0	3.5	0.6	1.8	4.4	7.5	3.5	26.8	3.3	51.4
1915.....	25.3	3.7	0.4	2.0	5.2	5.4	4.7	24.9	3.8	50.1
1916.....	25.9	4.5	0.4	2.6	4.7	7.3	3.8	24.2	3.2	50.7
1917.....	25.7	3.0	0.3	2.0	4.1	6.2	4.2	24.6	3.8	48.2
1918.....	23.4	5.0	0.6	2.0	4.3	3.2	3.2	27.2	2.9	48.4
1919.....	21.4	1.7	0.4	1.9	4.4	4.1	4.6	25.2	3.0	45.3
1920.....	25.1	4.4	0.4	1.7	5.2	4.3	4.5	27.2	2.9	50.6
1921.....	23.3	2.0	0.5	2.1	5.2	4.7	5.0	26.2	2.1	47.8
1922.....	23.2	1.2	0.3	1.4	3.7	3.3	4.7	24.8	2.5	41.9
1923.....	21.9	2.1	0.4	1.4	4.1	2.8	5.6	24.5	2.9	43.8

NOTE. The decrease for tuberculosis and the increase for malformations as shown in the above table are probably somewhat less than suggested by the figures, by reason of the fact that in the earlier years covered by the table the former heading included all deaths from hydrocephalus, which has more recently been included among the malformations. A proportion of the deaths attributed to hydrocephalus in the earlier years was also no doubt due to meningitis.

infant mortality are also no doubt due to the activities of the Infant Life Protection Branch of the Education Department, which supervises all children under six years of age who are placed privately by their parents or guardians in foster homes for payment, comprising for the



most part the illegitimate offspring of young single girls. The work is carried out under the supervision of trained nurses who are fully qualified in the care and feeding of infants and young children. In many cases the infants have been so neglected prior to their being placed in foster homes that the greatest care is necessary for their welfare.

In addition to the factors already considered, it should be noted there are local conditions in operation in New Zealand which are specially conducive to a high average standard of health and therefore to a low death-rate. Not only at ages under one year, but at virtually all ages, the rate of mortality is lower in New Zealand than in any other country.[1] The favorable local conditions are partly climatic, partly economic, partly social, partly racial (the great distance of New Zealand from the source whence her population has been recruited has acted as a selective agency), partly connected with the quality of the medical and health services, and partly the result of New Zealand's geographic isolation and consequent partial immunity from world-wide epidemics. I have elsewhere considered this matter rather more fully.[2]

Table III shows that the great fall in the rates of infantile mortality has been accomplished solely in connection with diseases, etc. which are not essentially pre-natal in origin. The decreases are most marked in the case of tuberculosis (partly fictitious—see footnote to Table), gastric and intestinal diseases, convulsions (possibly also partly fictitious since convulsions are normally merely symptoms of some other disease, and there is some reason for believing that there has been an increasing tendency for medical men to search for such other underlying diseases in returning the causes of death), epidemic diseases, and respiratory diseases, in that order. As might have been expected, deaths from epidemic diseases exhibit more violent annual fluctuations than do deaths from other causes. Of late years, the peaks that normally accompany such epidemics have been rising to an ever lessening degree. Broadly speaking, one might say that there has been a progressive decline in the relative importance of preventable causes of death.

If the decline in the rates of infantile mortality has been wholly confined to cases arising from other than pre-natal influences, we should expect the declines to be most marked in the later months of the first year of life, while during the first month of life in particular we should expect to find little or no secular decline. These expectations are strikingly borne out by the contents of Table IV. The secular decline is greater the older the age-group considered. It is marked except for infants dying under one month. A very large proportion of infants dying under one month, and a smaller proportion of those dying at a later age, are undoubtedly such that they have not really been born in a viable condition—they are virtually foredoomed to perish. Deaths at later ages are more preventable, and at such ages what has

[1] Sir Arthur Newsholme, *Elements of Vital Statistics*, Allen and Unwin, 1923, p. 211.
[2] E. P. Neale, *op. cit.*, pp. 226–227.

been said by Dr. F. Truby King, director of Child Welfare in New Zealand, is probably pretty near the mark—"in general babies don't die—they are killed"—not deliberately, of course; not from any lack of parental love; but from sheer lack of sound parental knowledge.

TABLE IV

DEATHS AT VARIOUS AGES UNDER ONE YEAR, BY SEXES, PER 1,000 CHILDREN OF EACH SEX BORN ALIVE

Year	Ages at Death in Complete Months									
	Under 1		1 and under 3		3 and under 6		6 and under 12		Total	
	M	F	M	F	M	F	M	F	M	F
1872	31	27	23	16	25	18	33	27	112	88
1873	40	27	22	22	23	22	31	29	116	100
1874	35	29	24	21	22	22	32	32	113	104
1875	45	31	25	21	30	24	37	37	137	113
1876	39	32	19	16	26	21	28	26	112	95
1877	34	25	18	16	20	19	27	22	99	82
1878	30	23	17	12	17	16	20	15	84	66
1879	39	30	25	20	23	19	32	27	119	96
1880	30	26	20	19	22	18	28	24	100	87
1881	44	33	26	19	29	23	37	33	136	108
1882	33	27	19	16	18	15	26	22	96	80
1883	33	29	20	20	26	21	31	28	110	98
1884	32	26	16	15	17	14	21	17	86	72
1885	33	28	18	14	21	16	25	23	97	81
1886	33	25	21	19	21	22	29	27	104	93
1887	33	23	19	17	23	19	26	27	101	86
1888	28	21	16	12	15	15	19	15	78	63
1889	31	26	16	15	18	17	19	16	84	74
1890	29	26	16	12	18	14	20	21	83	73
1891	35	25	20	20	22	16	23	21	100	82
1892	36	26	20	15	21	17	23	20	100	78
1893	34	25	21	13	17	17	27	21	99	76
1894	34	26	17	14	18	17	20	16	89	73
1895	34	28	18	18	19	16	23	20	94	82
1896	34	27	12	13	19	13	21	15	86	68
1897	30	24	13	13	16	13	20	15	79	65
1898	34	27	16	14	19	13	20	16	89	70
1899	38	27	22	19	22	19	22	22	104	87
1900	34	28	16	14	16	13	15	14	81	69
1901	34	26	14	13	16	12	15	13	79	64
1902	36	29	17	16	16	14	20	17	89	76
1903	35	29	18	13	17	17	17	16	87	75
1904	33	25	12	11	19	12	15	14	79	62
1905	33	26	13	11	14	11	14	12	74	60
1906	33	26	12	8	12	8	13	12	70	54
1907	34	27	21	14	20	20	22	19	97	80
1908	36	26	10	9	15	11	14	14	75	60
1909	35	25	12	9	11	8	13	10	71	52
1910	34	26	13	10	13	13	14	12	74	61
1911	33	23	10	7	10	8	11	11	64	49
1912	34	27	8	6	8	6	8	6	58	45
1913	34	25	11	10	10	7	10	11	65	53
1914	32	25	9	8	7	7	9	6	57	46
1915	35	23	7	6	7	6	8	7	57	42
1916	30	24	9	6	9	7	9	8	57	45
1917	32	24	7	6	7	6	8	7	54	43
1918	29	24	10	7	6	4	9	8	54	43
1919	33	23	7	6	5	5	6	6	51	40
1920	34	27	8	7	6	5	7	7	55	46
1921	33	28	8	5	6	4	6	5	53	42
1922	32	22	7	5	4	3	5	5	48	35
1923	33	25	8	6	3	3	6	4	50	38

The figures suggest that there has always been in the past, and that there probably still is, some preventable waste of infant lives.

The male rates of infant mortality are noticeably and without exception higher than the corresponding female rates, this being in conformity with conditions prevailing at practically all older age-groups. The percentage of the female rate of infant mortality to the male rate is almost invariably higher when deaths under twelve months are considered than when deaths under one month alone are taken into account. This shows that while male infants are throughout more likely to die than are female infants, this is specially true as regards neo-natal mortality (deaths under one month). These facts are well borne out by Table V. There is also evidence, both in this table and in Table IV that the secular decline in infant mortality has been more marked among female than among male infants.

TABLE V

PERCENTAGE OF FEMALE RATE OF INFANT MORTALITY TO MALE RATE

Year	Under 1 month	Under 12 months	Year	Under 1 month	Under 12 months
1872.....	87	79	1898.....	79	78
1873.....	68	86	1899.....	70	83
1874.....	83	92	1900.....	81	85
1875.....	69	82	1901.....	76	81
1876.....	82	85	1902.....	80	85
1877.....	74	83	1903.....	81	86
1878.....	75	79	1904.....	74	78
1879.....	76	81	1905.....	79	81
1880.....	87	87	1906.....	77	77
1881.....	75	80	1907.....	81	83
1882.....	82	83	1908.....	74	80
1883.....	88	89	1909.....	72	74
1884.....	80	84	1910.....	76	83
1885.....	84	84	1911.....	72	77
1886.....	75	89	1912.....	79	78
1887.....	69	85	1913.....	75	81
1888.....	75	81	1914.....	79	80
1889.....	84	88	1915.....	65	74
1890.....	92	88	1916.....	82	79
1891.....	71	78	1917.....	76	80
1892.....	72	78	1918.....	83	80
1893.....	74	77	1919.....	71	79
1894.....	77	82	1920.....	79	84
1895.....	82	87	1921.....	84	80
1896.....	81	80	1922.....	72	73
1897.....	81	83	1923.....	75	76

In many countries it has been observed that whooping cough, almost alone among all diseases, has been responsible for more deaths of girl babies than of boy babies,[1] and it has been suggested that the higher mortality generally of both male infants and foeti is due to the fact that the male possesses a different complex of chromosomes, the cells of the male body being lacking in chromatin: the different conformation of the male larynx might account for the exception that exists in

[1] Sir Arthur Newsholme, *op. cit.*, p. 357.

connection with whooping cough.[1] In New Zealand, however, male deaths from whooping cough normally exceed female deaths from whooping cough amongst infants (except infants under one month), though the excess is much smaller than that found in connection with most other diseases. On the other hand, measles would appear to claim more girl babies than boy babies in New Zealand. These facts will be apparent from a study of Tables VI and VII, in which the number of deaths, by causes, sexes, and ages, during the past four years are set against the number during the years 1904 to 1907, the last four years prior to the great fall in New Zealand's infant mortality. Of course in some cases the figures are small, and where this is so, too great weight must not be attached to the conclusions suggested.

The rates of infant mortality have usually been from 1½ to 2 times as high for illegitimates as for legitimates, the figures for 1921 to 1923 having been: 1921—males 106, females 81; 1922—males 83, females 56; 1923—males 72, females 57. The average for both sexes over the three years was 76, of which 50 were accounted for by malformations and early infancy combined. It would therefore appear that the infant mortality for illegitimates is almost equally high, relative to the mortality for legitimates from both pre-natal and post-natal causes. Unfortunately, data are not available far enough back to indicate whether or not illegitimate infant mortality has declined as rapidly as legitimate infant mortality. There is little evidence of any significant difference in New Zealand between the sex ratio at death of legitimate and illegitimate infants such as has been noticed elsewhere.[2] The numbers dealt with in New Zealand, however, are small (286 deaths of illegitimate infants during the three years above considered).

Statistics are available in New Zealand as to the months in which deaths occurred in the case of infants under 5 years of age. The figures for 1923 are shown in Table VIII.

The range between maximum and minimum monthly figures is much greater relative to the monthly average for males than for females, suggesting that seasonal influences are more important in the case of male infant mortality than in that of female. In most countries a double maximum has been observed, one in winter associated with respiratory diseases, and one in summer associated with diarrhoea and allied diseases.[3] There is no evidence in the table of the existence of even a tendency towards a second maximum in summer under present day conditions in New Zealand. (Unfortunately, comparable data

[1] S. J. Holmes, *Evolution and Eugenics*, Routledge, 1923, Ch. VII.
[2] Sir Arthur Newsholme, *op. cit.*, p. 357.
[3] I. S. Falk, *Principles of Vital Statistics*, Saunders, 1917, p. 85.

TABLES VI AND VII
NUMBER OF INFANTS, ACCORDING TO SEX AND AGE, DYING FROM CERTAIN PRINCIPAL CAUSES
(A) 1904 TO 1907

TABLES VI AND VII

NUMBER OF INFANTS, ACCORDING TO SEX AND AGE, DYING FROM CERTAIN PRINCIPAL CAUSES

(A) 1904 TO 1907

Cause	Males					Females				
	Under 1 month	1 and under 3 months	3 and under 6 months	6 and under 12 months	Total	Under 1 month	1 and under 3 months	3 and under 6 months	6 and under 12 months	Total
Measles	1	2	7	7	17	1		3	13	17
Whooping Cough	7	43	33	46	129	12	30	42	40	124
Diphtheria				5	5				2	2
Influenza	2	10	3	6	21	1	4	3	5	13
Syphilis	8	9	11	1	29	6	10	7	1	24
Convulsions	90	35	40	49	214	74	21	16	40	151
Bronchitis, Pneumonia, etc.	56	129	116	162	463	41	76	85	130	332
Diarrhoea, Enteritis, etc.	62	184	304	226	776	53	139	219	202	613
Hernia, Intestinal Obstruction, etc.	16	5	10	22	53	6		6	8	20
Congenital Debility and Malformation, Icterus	422	156	108	60	746	300	104	101	56	561
Premature Birth	697	40	11	3	751	541	46	6	3	596
Total Deaths, all causes	1,644	719	791	787	3,941	1,219	519	599	671	3,008

(B) 1920 TO 1923

Cause	Males					Females				
	Under 1 month	1 and under 3 months	3 and under 6 months	6 and under 12 months	Total	Under 1 month	1 and under 3 months	3 and under 6 months	6 and under 12 months	Total
Measles			3	11	14	3	2	1	13	19
Whooping Cough	1	16	16	22	55	3	22	12	16	53
Diphtheria		1		3	4		2		5	7
Influenza	5	3	2	6	16	2		3	9	14
Syphilis	9	5	6		20	3	6	2	2	13
Convulsions	72	17	12	17	118	41	10	11	13	75
Bronchitis, Pneumonia, etc.	54	78	46	101	279	23	55	43	82	203
Diarrhoea, Enteritis, etc.	33	65	56	54	208	11	33	42	41	127
Hernia, Intestinal Obstruction, etc.	4	2	8	12	26	3	4	8	4	19
Congenital Debility and Malformation, Icterus	510	160	63	46	779	391	86	48	37	562
Premature Birth	883	47	6	1	937	654	48	7		709
Total Deaths, all causes	1,963	461	274	360	3,060*	1,436	320	202	301	2,259

*Includes two murdered male infants of unknown age.

for years when in the past infant mortality from diarrhoea, etc., was high, are not available.) It is just possible, of course, that in the table the existence of a summer maximum of a secondary character in the case of infants under one year is masked by the existence of a summer minimum in the case of infants one and under five years. (In 1923, 720 males and 505 females died under one and 213 males and 182 females at one and under five.) The fact that, if such a secondary summer maximum exists, it is small enough to be so masked, is however in itself a significant one.

TABLE VIII

DEATHS OF INFANTS UNDER FIVE YEARS ACCORDING TO SEX AND BY MONTHS IN WHICH OCCURRING, 1923

Month of death	Male deaths	Female deaths	Total deaths
January	56	40	96
February	51	46	97
March	62	46	108
April	64	54	118
May	71	47	118
June	75	73	148
July	105	72	177
August	123	79	202
September	86	67	153
October	90	66	156
November *	75	50	125
December *	75	47	122
	933	687	1,620

* Includes an allowance for births occurring during these months, but not registered until the following year.

The four most populous provinces of New Zealand, proceeding from north to south, are Auckland, Wellington, Canterbury, and Otago (including the Southland subdivision). New Zealand being in the Southern Hemisphere, the most northerly of these is the warmest. Auckland stretches from latitude 34½ degrees south to 39 degrees (with its greatest density of population in Auckland City at latitude 37 degrees); Wellington from 39 degrees to 41½ degrees (with its greatest density in Wellington City, the Capital, at 41½ degrees); Canterbury from 43 degrees to 45 degrees (with its greatest density in Christchurch City at 43½ degrees); and Otago from 45 degrees to 47 degrees (with its greatest density in Dunedin City, at 46 degrees). The infant mortality rate from diarrhoea and enteritis, as indeed might have been expected, increases regularly from south to north, the figures for the years 1920 to 1923 per 1,000 children born alive having been 1.4 in Otago, 2.4 in Canterbury, 3.4 in Wellington, and 3.7 in Auckland. Except for the fact that Wellington is in each case abnormally high, the death-rates of infants per 1,000 children born, due to bronchitis, pneumonia, and pleurisy, on the contrary increase as one goes from

the warmer north to the colder south, the figures being 2.6 in Auckland, 4.2 in Wellington, 3.1 in Canterbury, and 3.9 in Otago. (The abnormal position of Wellington may be partly due to the fact that city housing conditions there are notoriously bad.)

The infant mortality rates are higher in the cities than in the country districts. On this point interesting light is shed by Table IX.

TABLE IX

RATES OF INFANT MORTALITY ACCORDING TO LOCALITY, EACH YEAR 1917 TO 1923

	1917	1918	1919	1920	1921	1922	1923
Four Primary Centers:							
Auckland	61	58	49	48	54	44	44
Wellington	57	71	60	74	61	49	47
Christchurch	49	57	52	71	54	54	66
Dunedin	40	46	45	59	47	40	43
Total	54	58	52	62	55	47	50
Eight Secondary Centers	55	52	47	54	62	44	51
Twelve Urban Areas (Total)	54	57	51	60	56	46	50
Total for Dominion	48	48	45	51	48	42	44

Dunedin's consistently low position as compared with Christchurch may be due partly to the fact that Dunedin was the home of the Plunket movement, partly to the fact that Dunedin is a rather small city. The fact that Wellington has almost always possessed a higher infant mortality rate than the four primary centers as a whole may be traced to the bad housing conditions already referred to: the census of 1921 revealed that 13.62 per cent of the population of Wellington City was residing in dwellings which contained more than 1½ persons per room: the next highest figure for a primary urban area was in Auckland, with 11.97 per cent of her population so housed. (The 1921 census populations of the four primary centers were: Auckland 157,757; Wellington 107,488; Christchurch 105,670; and Dunedin 72,255. The largest of the secondary centers had a population of 23,523, and the smallest 10,632.) The higher infantile mortality of the cities is of course in conformity with the general experience in other lands.

A STUDY OF NET RAILWAY OPERATING INCOME

By Leroy E. Peabody

Net railway operating income is the return to bond and stockholders in railway companies. During the years 1895 to 1923, nearly three decades, this return has varied widely both in amount and as a percentage of total operating revenues. The inclusion in railway net of interest for bondholders and dividends for stockholders is in contrast to the net income of the ordinary industrial corporation, which considers as "profits" only the return to stockholders and appropriations for surplus and betterments. The Transportation Act of 1920 fixes 5¾ per cent as a proper return upon investments in railway property. The rates of return actually realized by the railroads in three of the years which have elapsed since the termination of federal control have been:

	Per cent
1921	3.31
1922	4.14
1923	5.10

Two sharply contrasting periods in the history of railway net are easily discernible. The first, from 1895 to 1916, is comparatively normal. Traffic growth was remarkably steady and there were few strikes or disturbances of national importance. Total operating revenues, compensation to employees, amounts spent for materials and nearly all items of expense grew steadily as the following analysis shows.

The second period, 1916 to 1923, included the unusual traffic increases of the war and post-war years, the unprecedented traffic decline of 1921, the strikes in the coal and railway industries of 1919, the switchmen's strike of April 1920, the shopmen's strike of July 1922 and the coal strike of April 1922. These disturbances involved large numbers of men and were national in character. The abnormal fluctuations of railway activities in the second period make comparison difficult and generalizations unsafe.

During 1895 to 1916 increases in railway net correspond closely to increases in traffic. So long as the railways could get increased traffic, their profits increased. During the second period the amounts expended for compensation and materials are the controlling factors and, despite phenomenal traffic, railway net declined until in 1920 it was

but 0.28 per cent of total operating revenues and amounted to $17,000,000 as compared with $628,000,000, the average per year from 1895 to 1923 inclusive.

In the first period railway net averaged 4.77 per cent of cost of road and equipment, while in the second period the corresponding average fell to 3.45 per cent. Taxes and fuel costs have risen in nearly every year throughout both periods.

Probably the greatest contrast between the two periods is in the relationship of railway net to compensation to employees (Chart V). During the first period these two items moved together. During the second period the movements were in opposite directions except from 1922 to 1923.

1895–1916

Railway net is the difference between railway receipts and railway expenditures.

Railway receipts depend upon volume of traffic and the level of rates. Railway expenditures are determined largely by the rate of compensation (times the number of employees) and the cost of materials and locomotive fuel. On the average these expenses are 68.2 per cent of total revenues. The number of employees fluctuates with the volume of traffic; although in periods of increases in traffic, the increase in number of employees is in smaller proportion, while in periods of decreases in traffic, the number of employees decreases less rapidly than the decrease in traffic. During the period chosen (1895–1916) this factor would operate to increase profits, since traffic increases are very uniform throughout the period.[1] To offset this factor we have decreases in rates (measured by the average revenue per ton-mile) of about 16 per cent.

The following table will show the relative importance of the various items to total operating revenues:

	Average 1895-1916 (millions of dollars)	*Per cent of total operating revenue*
Compensation to employees	897	41.5
Materials, etc.	420	19.4
Fuel	158	7.3
Taxes	77	3.6
Net railway operating income	610	28.2

The period 1895 to 1916 is chosen since the years prior to 1895 show the effects of the depression of the early nineties. The trends during these earlier years were downward, in contrast to a continuous upward

[1] A straight line trend fits revenue ton-miles for the period very closely. With the exception of seven years, the percentage deviation of the actuals from the straight line is less than 6 per cent. The greatest deviation is 10 per cent.

trend for all items during the selected period. An additional reason for the choice lies in the fact that compensation to employees is not available before 1895 and that I. C. C. reports were changed from fiscal to calendar year in 1916. After 1916 the increases in traffic, compensation, and many other of the items are so great that the later years are not properly comparable with the period 1895–1916.

Total operating revenues and expenses for the period of years 1895 to 1916, inclusive, are given in Table I. The expenses are divided between compensation to employees, locomotive fuel, taxes, and materials. Materials include loss and damage, injuries to persons, insurance, and depreciation charges. These latter items are about one-fifth as great as total expenditures under this head. The balance between revenues and expenses is net railway operating income.

An examination of the figures of Table I shows that these items may all be properly fitted with a linear trend. The percentage variations from the line of trend is greater for net railway operating income than for any of the other items.

The equations of the lines of trend are for:

Compensation to employees	$y=24.932\,t+897.091$
Materials	$y=12.590\,t+420.227$
Fuel	$y=\ 4.702\,t+157.773$
Taxes	$y=\ 2.698\,t+\ 76.500$
Net railway operating income	$y=12.712\,t+610.000$
Revenue ton-miles	$y=\ 5.423\,t+198.454$

In the first five equations y represents the item in millions of dollars, t for the time in half-years, with the origin halfway between 1905 and 1906. In the last equation y represents revenue ton-miles in billions.

The average advance per year for any item throughout the period 1895–1916 will be given by doubling the coefficient of t in the equation for that item. For compensation to employees the average annual gain experienced was $49,864,000. The mean compensation for the period was $897,091,000. The annual percentage gain was then 5.6. The items with their average annual percentage gains may be compared in the following table:

	Per cent
Compensation to employees	5.6
Materials, etc.	6.0
Fuel	6.0
Taxes	7.1
Net railway operating income	4.2
Revenue ton-miles	5.5

The annual percentage gains for all items except taxes and net railway operating income were close to 6. Taxes increased proportionately more than the others, while railway net increased considerably less

TABLE I

	Revenue ton-miles (millions)	Total operating revenues	Compensation to employees	Locomotive fuel	Other expenditures (materials, etc.)	Railway tax accruals	Net railway operating revenue
ALL STEAM ROADS:							
Year ended							
June 30, 1895	85,228	$1,075,371,462	$445,598,261	$70,271,745	$209,940,409	$38,146,320	$311,504,727
1896	95,328	1,150,169,376	468,824,531	69,786,920	234,377,593	37,961,655	339,218,677
1897	95,139	1,122,089,773	465,601,581	65,044,670	221,878,513	41,119,490	328,445,519
1898	114,078	1,247,325,621	495,055,618	72,469,777	250,447,881	41,928,719	387,423,626
1899	123,667	1,313,610,118	522,967,896	77,187,344	256,813,759	44,396,669	412,244,450
1900	141,597	1,487,044,814	577,264,841	90,593,965	293,569,805	44,445,145	481,171,158
1901	147,077	1,588,526,037	610,713,701	104,926,568	314,757,001	46,707,835	511,420,932
1902	157,289	1,726,380,267	676,028,592	120,074,192	320,145,963	50,054,136	560,077,384
1903	173,221	1,900,846,907	a775,943,415	146,509,031	335,086,406	53,251,852	590,056,203
1904	174,522	1,975,174,091	817,598,810	158,948,886	362,348,557	56,801,756	579,476,082
1905	186,463	2,082,482,406	839,944,680	156,429,245	394,228,227	58,712,179	633,168,075
1906	215,878	2,325,765,167	b932,429,653	170,499,133	433,948,485	69,064,057	719,823,839
1907	236,601	2,589,105,578	1,072,386,427	200,261,975	475,867,412	73,743,473	766,846,291
1908	218,382	2,440,638,832	1,035,437,528	201,905,054	473,059,209	78,673,794	634,794,284
1909	218,803	2,473,205,301	988,323,694	188,735,868	472,974,642	85,139,554	710,474,052
1910	255,017	2,812,141,575	1,143,725,306	217,780,953	520,372,859	98,034,593	805,097,145
1911	253,784	2,852,854,721	1,208,466,470	c222,302,419	545,562,975	102,657,157	744,669,102
1912	264,081	2,906,415,869	1,252,347,697	c224,516,528	558,203,304	113,818,605	727,458,036
1913	301,730	3,208,647,370	1,381,334,368	c241,598,314	626,345,255	d122,005,424	806,800,960
1914	288,637	3,126,520,234	1,381,117,292	c235,231,481	663,059,713	d139,591,520	674,623,250
1915	277,135	2,956,193,202	1,242,319,254	c208,968,991	637,394,711	137,775,259	694,276,111
1916	343,477	3,472,641,941	1,403,968,437	c226,880,054	646,353,737	150,014,541	1,002,934,791

a Includes estimate of $18,622,000 compensation for C., M & St. P. not reported to I. C. C.
b Includes estimate of $31,628,000 compensation for So. Pac. not reported to I. C. C., records destroyed by fire.
c Class I roads only.
d Class I and II roads only.

NOTE.—Data for years prior to 1908 inclusive, switching and terminal companies. An average of seven per cent of compensation to employees is properly chargeable to capital. Materials is the difference between total operating revenue and expenses (including net railway operating income); these capital charges would therefore have the effect of adding to materials what would be deducted from compensation. As homogeneous, comparative figures were desired, these changes were not made. Equipment and joint facility net rentals included in revenues and expenses up to 1908.

than the other items. In what follows then, it should be kept in mind that during the whole period under discussion, railway net has increased less rapidly than other items of expense, especially during the latter half of the period.

Chart I shows the ratios of the actual values to the trend values for net railway operating income, compensation to employees, and materials from 1895 to 1916. A measure of the year to year changes in these items relative to each other after allowance for normal growth, is desired; especially the changes in the items of expense relative to those of net railway operating income. Since the trends are continuously upward for all items, changes in the items of expense relative to net income must take into account the average annual increase in the several items.

To compare the changes in compensation and materials with the changes in railway net, the trend ratios of compensation and of materials have been divided by the trend ratios of railway net (these ratios are those of Chart I). In equation form we may write:

$$\frac{\text{Compensation for any year}}{\text{Railway net for any year}} = (Z)\frac{\text{Normal compensation for any year}}{\text{Normal railway net for any year}}$$

The coefficient of Z in the right-hand member of this equation is fixed by the movement of compensation and railway net throughout the period of years taken for a comparison of these items. This coefficient changes slightly from year to year, but the change is uniform and always in the same direction. The values of Z are thus a measure of the relative changes in the several items from year to year. They are plotted in Chart II.

From Chart II it is seen that from 1895 to 1900 materials and compensation increased more slowly than railway net. The relations between the items for any part of the period 1895 to 1916 may be read from Chart II.

These relations may be shown more clearly by making the comparison between compensation, materials, railway net and total operating revenue, using the method just outlined. In Chart III the relations of these items to total operating revenue are shown. Railway net moves in the opposite direction to that of compensation and materials.

From 1897 to 1898 net moved up 9 points, compensation down 5 points and materials down one point. If net be weighted by 3, compensation by 4 and materials by 2 (weights roughly proportional to percentages of the respective items to total operating revenue), the movement upward of net is 27; the movement downward of compensation and materials is 22 points. Again from 1910 to 1911 net moved

down 9 points, compensation up 4 points, materials up 3 points. The weighted movements are: net, downward 27 points; compensation and materials, upward 22 points.

CHART I.

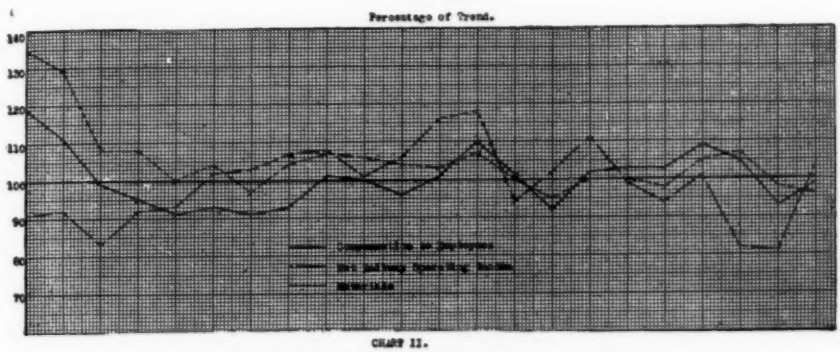


CHART II.

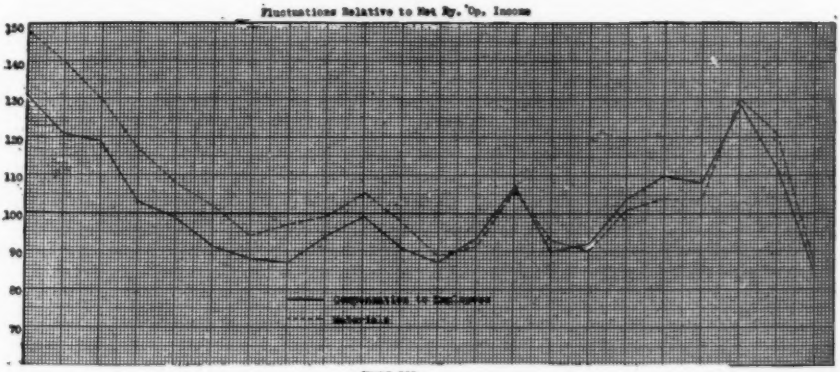
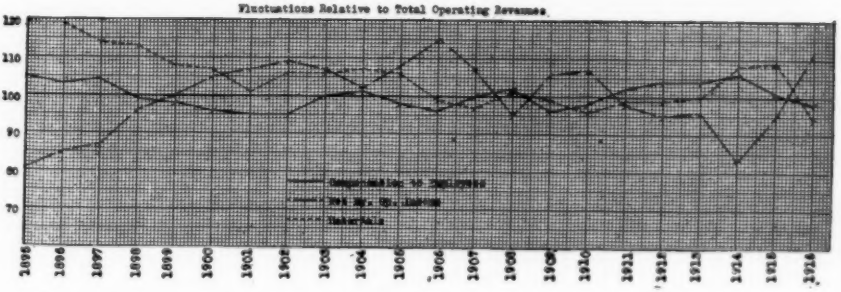


CHART III.



We may use Chart III to gauge the short movements and split the 1895–1916 period into seven parts. From 1895 to 1902, 1904 to 1906, 1908 to 1910, and 1914 to 1916 net increased more rapidly relative to total operating revenue than compensation and materials. In the remaining years the reverse is true.

From the figures of Table I the following annual percentage increases are found:

Period	Annual percentage increases in			Revenue ton-miles
	Railway net	Compensation	Materials	
1895–1902	11.3	7.4	7.5	12.1
1902–1904	1.7	10.5	6.6	5.7
1904–1906	12.2	7.0	10.0	11.7
1906–1908	−5.9	5.6	4.5	0.5
1908–1910	13.4	5.3	5.0	8.5
1910–1914	−4.1	5.2	6.9	3.3
1914–1916	24.3	0.9	−1.3	9.4

During the periods of increase in railway net the increase in traffic corresponds very closely except for the period 1914–1916. Evidently the large increase in net, 24.3 per cent annually, during this period was caused in part by the increase in traffic but even more by the slight increase in compensation, 0.9 per cent annually, and the decrease in materials, 1.3 per cent annually. The average ratio of railway net to total operating revenue for the periods of high net is 29.88 per cent; for the periods of low net it is 26.80 per cent. Evidently Chart III is an efficient measure of the relationships of railway net, compensation and materials. Net is most dependent upon traffic; but it may increase in a greater ratio than traffic if compensation and materials decrease, or if they increase but slightly, as in the period 1914–1916.

The short-time movements of net have just been shown, while the long-time movements were discussed earlier. Since traffic increased continuously from 1895 to 1916 and "fixed expenses" increased but slightly in proportion to the increase in traffic, it might be expected that net would greatly increase during this period. That it did not may be attributed to the facts that cost of materials and compensation to employees increased even more rapidly than traffic (see page 370), and that rates (measured by the average revenue per ton-mile) decreased throughout the period. Had the revenue per ton-mile remained constant from 1895 to 1916, net would have been about 50 per cent higher, despite the advances in wages and materials. If the 1895 revenue per ton-mile had been received for the 1916 traffic, railway net would have been $1,415,682,820 or 41.15 per cent greater than it was for that year.

1916–1923

Probably the best view of the differences between the periods 1895–1916 and 1916–1923 may be gained from a comparison of the annual

TABLE II

	Year ended December 31								
	1916	1917	1918	1919	1920	1921	1922	1923
CLASS I ROADS:								
Total operating revenues	$3,596,865,766	$4,014,142,747	$4,880,953,480	$5,144,795,154	$6,178,438,459	$5,516,598,242	$5,559,092,708	$6,290,153,622
Compensation to employees	1,468,576,394	1,739,482,142	2,613,813,351	2,843,128,432	3,081,801,193	2,765,218,079	2,640,738,182	3,004,659,673
Locomotive fuel	250,544,862	373,929,538	500,225,205	474,174,792	674,836,361	523,724,145	518,283,929	520,000,000
Other expenses	638,277,156	695,913,444	868,029,641	1,082,412,291	1,470,953,592	1,273,726,078	1,255,500,223	1,371,587,410
Taxes	157,113,372	213,920,095	223,175,379	232,601,396	272,061,453	275,875,990	301,034,923	332,057,588
Net railway operating income	1,040,084,517	934,068,770	638,568,603	454,984,953	17,226,902	600,937,356	760,187,305	961,236,657
Revenue ton-miles (millions)	362,444	394,465	405,379	364,263	410,306	306,840	339,285	413,562

averages of the distributions of total operating revenue in the two periods.

	Average (millions of dollars)		Per cent of total operating revenue	
	1895–1916	1916–1923[1]	1895–1916	1916–1923
Compensation to employees	897	2,595	41.5	51.0
Materials	420	1,082	19.4	21.3
Fuel	158	482	7.3	9.5
Taxes	77	251	3.6	4.9
Net railway operating income	610	676	28.2	13.3
Cost of road and equipment	12,783	19,610		

[1] The figures are for the calendar years 1916 to 1923 inclusive for Class I roads, which earn 96 per cent of the total revenues, and are computed from the items in Table II.

While for 1916–1923 average net railway operating income, or profits, increased 12.5 per cent they were but 13 per cent of the total operating revenue, as against 28 per cent in the earlier period. Railway net was 4.77 per cent of cost of road and equipment in the earlier period, 3.45 per cent of cost of road and equipment in the later period. The average amounts spent for labor and salaries rose 189 per cent and this item required the expenditure of 51 cents out of each dollar of operating revenue, as against 41.5 cents in the earlier period. Materials increased 158 per cent on the average and demanded 21.3 cents out of each revenue dollar for the later period, as compared with 19.4 cents during 1895–1916. Fuel increased 205 per cent while taxes increased 226 per cent. Fuel and taxes thus showed the greatest *rate* of increase, while labor and materials were responsible for the greatest *amounts* of increase.

These items, except for taxes, increased rapidly until 1920, the peak year, after which there was a sharp drop in 1921 and a gradual rise from that time. Taxes rose evenly throughout the whole period and are the only item that might be said to have a distinct trend, although it is worth noting that total operating revenue for 1923 was higher than the 1920 peak. Taxes were the only other item that exhibited a greater value for 1923 than for 1920.

Since it is not feasible to fit any of the items, except taxes and possibly total operating revenue, with trend lines, we will modify the analysis used in the first period by using the average of the period as a base, instead of the linear trend. Chart IV for the second period will correspond to Chart III for the first period with the exception just noted. The figures of Table II show the following:

Period	Annual percentage increases in			Revenue ton-miles
	Railway net	Compensation	Materials	
1916–1920	−24.6	37.7	32.6	3.3
1920–1923	597.4	−6.2	−2.3	0.2

These figures may be compared with those upon page 374. Instead of the close agreement between net and traffic found in the earlier period, for 1916–1923 traffic and net did not move together. The amounts expended for compensation and materials were the controlling factors from 1916–1923 (see Chart V); while from 1895 to 1916 increases in traffic appear to be the factors that controlled railway net. The last statement summarizes the differences between the two periods. It is true that increased taxes and expenditures for locomotive fuel helped toward this end, but the extraordinary increases in the rates of compensation and prices of materials were the main factors.

Net railway operating income in 1920 was 0.28 per cent of the total operating revenue for that year. The decline in net from 1916 to 1920 was much more rapid than the rise from 1920 to 1923 (see Chart IV) if the growth in total operating revenue is taken into account. The decline in amount of net from 1916 to 1920 was less than the increase in amount of net from 1920 to 1923 (Table II).

The year 1920 was the year of least net in railway history since 1895. It was the year of greatest expenditure for labor, materials, and fuel. Total operating revenue in 1920 was distributed as follows:

	Millions of dollars	*Per cent of total operating revenue*
Labor	3,681	59.59
Materials	1,471	23.81
Fuel	675	10.92
Taxes	272	4.40
Net railway operating income	17	0.28

The percentage distribution may be compared with those for the two periods, given on page 376.

Total operating revenue for 1923 was higher than that for 1920 and was distributed much more favorably from the standpoint of railway profits. Relative to total operating revenue all the items of expense in the distribution have fallen since 1920. Compensation to employees declined most and quickest, although this item increased from 1922 to 1923 (see Chart IV). Railway fuel costs fell nearly as rapidly as compensation, and continued to fall during 1922 and 1923. Expendi-

tures for railway materials fell (relative to total operating revenue) at very nearly a constant rate. Taxes continued to rise (relative to total operating revenue) until 1922 and fell but slightly from 1922 to 1923.

It is evident that railway returns are small as compared with the returns of other industrial undertakings. Treasury Reports [1] for the year 1916 show an average return for 503 chemical and allied industries of 47.8 per cent, the median return being approximately 32 per cent. In 1917, 534 chemical and allied industries showed an average return upon capital stock of 49.3 per cent, or a return upon invested capital of 32.5 per cent. The median return upon capital stock was approximately 30 per cent for this year.

In the bituminous coal industry the following is shown:

	Number of corporations	Total net income	Total capital stock	Average return (per cent)
1916.....	346	$10,769,915	$41,589,652	26.0
1917.....	357	43,911,978	62,350,855	70.5

The average return upon invested capital for 1917, the only year for which the figures for invested capital were given, was 45.6 per cent. To be sure the report to the Treasury was required only of corporations earning 15 per cent or more upon their capital stock in the year 1917, yet railway returns, even including sums paid to bondholders, are far less than these.

[1] Corporate Earnings and Government Revenues, Treasury Dept., 1918.

A FURTHER DISCUSSION OF THE VARIABILITY OF FAMILY STRAINS IN THE NEGRO–WHITE POPULATION OF NEW YORK CITY [1]

By Melville J. Herskovits

In an investigation of the variability of a population, there are a number of sub-variabilities which can be studied, all of which must be taken into consideration when a study of the general variability of a population in a given trait is to be made. Thus, there is, besides the individual variability, that of the individuals themselves from day to day. In the case of growing children, this is an important factor, and is generally recognized where child-data are being worked with. But even with adults, there is the change which occurs with the day's activity, particularly in such matters as height, or weight. Again, a population is not made up of isolated individuals, but is composed of family lines, and the variability of these is important, as well as the variation within the families. Although great attention is usually paid to the differing variabilities of populations taken as a whole, these others have usually been overlooked.

The present paper is concerned with variations within the family strains, and between these strains, in the Negro-White population of New York City.[2] The measurements were taken at Public School 89, Manhattan, a school the pupils of which are about 98 per cent Negro. The amount of mixture which has gone into the racial composition of this population, is, of course, an unknown quantity, but there is strong reason to believe that an estimate of 15 to 20 per cent pure Negro would be high. In other words, the population is almost entirely mixed. By measuring the variability of family strains an indication may be had of the amount of mixture, and, at the same time, a measure of the variability within these family lines. The statistical procedure needed for such analysis was given by Boas in a paper published some time ago.[3] The writer also has prepared a paper [4] dealing with the

[1] A paper read at the meeting of Section H., American Association for the Advancement of Science, at Washington, D. C., January 2, 1925.

[2] This study has been carried on under a fellowship grant of the Board of Fellowships in the Biological Sciences, National Research Council. The writer is indebted to Dr. Jacob M. Ross, principal of the school where these measurements were taken, and his teaching staff, for their numerous courtesies to him, and to Prof. Franz Boas for statistical advice.

[3] Franz Boas, "On the Variety of Lines of Descent Represented in a Population," *American Anthropologist*, (N. S.) Vol. xviii, No. 1, 1916. pp. 1–9.

[4] "On the Negro-White Population of New York City; the Use of the Variability of Family Strains as an Index of Homogeneity or Heterogeneity," *Proceedings of the XXIe Congrès des Americanistes*, First part, The Hague, August 1924.

variability of the family strains of the same population under consideration here, and the variability within the family strains, based on the cephalic index, and the results were compared with Boas' results. It is unfortunate that figures for the traits used here have not been worked out for populations other than the one under consideration. Outside of the ones cited above, in which cephalic index was the trait chosen, there are no data which are comparable. The present results, however, give a basis for comparisons to be made in the future, and afford a cross-check to see if the figures arrived at for cephalic index for the Negro-White population of New York City are, in the main, proportionate for those of nose-breadth, interpupillary distance, the height of the ear, and the length of the middle finger, which are the traits dealt with.

The variability of family strains is chiefly useful as an indication of the homogeneity or the heterogeneity of a given population. If we suppose a population which has been inbred for many generations, the common ancestors will be largely the same, and we should expect to find a very small variability of the family strains, as each family line would have sprung from about the same stock as every other one; or, to put the matter in a slightly different fashion, any one family would adequately represent the total population being dealt with. On the other hand, when we turn to a consideration of a population such as that of a modern city in the United States, no one family would be representative. That is, the family lines would be so divergent that the variability would be much larger than in our inbred population, since the number of ancestors would be so much greater. We might think of each family as a small race and, indeed, the term would be really applicable, and perhaps more so than the use we make of it today, since it is highly problematical the extent to which the members of a race are organically related to one another. At the same time, we must not make the mistake of assuming that a low variability of the family strains of a population denotes that the population is racially pure. It is merely an indication of its homogeneity or heterogeneity, for if we had (as is actually the case among the Bastards of South Africa) a population springing from ancestors of two distinctive racial groups, isolated after the crossing and inbred through generations, the variability of the family strains would be very low, provided that there was no social selection present.[1]

The assumption is made that, to calculate the variability of the family strains, one must utilize the averages of the fraternities in the population which is being studied. Since this would be possible from

[1] F. Boas, *op. cit.* p. 2.

direct observation only if there were a very large number of children in each family, and since this is usually not the case, the variability must be calculated for families of different numbers of children separately on the theoretical assumption of an infinite number of children in each family for the figures obtained, and then, through the combination of these, the final results may be reached. And since working with children means that the growth factor is present, the averages and correlations must be made in terms of the deviations of each member of the fraternities from the general average for their respective ages in the population of which they form a part. If we call the general average of the measurement of the trait selected a, its standard deviation σ, the number of children in a given family m, and the deviation of each child from the general average x, then the average for each family will be

$$a = \frac{x_1 + x_2 + x_3 + \ldots + x_m}{m}$$

and the mean square variability of these averages will be

$$S^2{}_m = \frac{\sum \left(\frac{x_1 + x_2 + x_3 + \ldots + x_m}{m} \right)^2}{n_f}$$

where n_f is the total number of families.

Developing this formula, Boas derived he following formula for the variability of family strains for fraternities of any given number of children:

$$(1) \qquad S^2{}_m = \frac{1 + (m-1)r}{m} \sigma^2$$

where r is the coefficient of fraternal correlation.

However, if we assume an infinite number of children to each family, as we must to obtain the theoretical variability of the family strains and to eliminate the error due to the small number of children in the actual families, we get

$$(2) \qquad S^2 = r\sigma^2 = \frac{\Sigma xx}{n_f}.$$

Therefore "the heterogeneity of descent may be determined directly from the two values for r and σ." [1]

In the computation of the coefficient of fraternal correlation, however, there is again the necessity for treating separately the families of two, three, four or more children. This arises from the fact that in

[1] F. Boas, *op. cit.* p. 3.

families of two children, each child is correlated only once, *i. e.*, we have child *a* times child *b*. In families of three, however, we have child *a* times child *b*, child *a* times child *c*, and child *b* times child *c*, each child being correlated twice. And so forth, each child being correlated $(m-1)$ times. Thus, the larger the number of children in a family, the greater will be the error in r, and the greater that in $r\sigma^2$. Therefore Boas has calculated the error in s^2, as follows:

$$\epsilon^2 = \frac{2}{m-1} \cdot \frac{1 + 2(m-2)r + (m^2-3m+3)r^2}{n} \sigma^4$$

in which n stands for the total number of individuals in the families having the given number of children m; and, from this, for families of two children,

$$\epsilon_2^2 = \frac{2}{1} \cdot \frac{1+r^2}{n} \sigma^4 \tag{3}$$

and for those of three children,

$$\epsilon_3^2 = \frac{2}{2} \cdot \frac{1+2r+3r^2}{n} \sigma^4. \tag{4}$$

With this error known, the proper weighting can easily be carried out, as will be shown.

To compute the variability of the families—that is, the variation within the family lines—all that need be known is the general mean square variation of the entire series, and the variability of the family lines. Boas remarks, "The square standard variation of individuals composing a family, *i. e.*, of the fraternities, is a correlate of the variability of family line, and must average $\sigma^2(r-1)$."[1] This is derived in the following manner: given an individual, a member of a family of an infinite number of children and a part of a general population, the deviation of that individual from the general mean, x, will be composed of his deviation from the average of his family, ξ, and the deviation of that average from that of the general mean, η. Or,

$$x = \xi + \eta$$

Then, the mean square deviation of these deviations, since the variability of the individual within the family and that of the family within the total population may be considered as independent phenomena, would be

$$\frac{\Sigma x^2}{N} = \frac{\Sigma \xi^2}{N} + \frac{\Sigma \eta^2}{N}.$$

[1] F. Boas, *op. cit.* p. 9.

But, $$\frac{\Sigma x^2}{N} = \sigma^2$$

while $$\frac{\Sigma \xi^2}{N} = s_f^2$$

and $$\frac{\Sigma \eta^2}{N} = s^2.$$

Therefore, $$\sigma^2 = s_f^2 + s^2$$

while (5) $$s_f^2 = \sigma^2 - s^2.$$

However, $$s^2 = r\sigma^2$$

therefore, $$s_f^2 = \sigma^2 - r\sigma^2$$

or, (6) $$= \sigma^2 (1-r)$$

So that, in the present paper, to get around the difficulty presented in reconciling the varying values of r for fraternities of varying numbers of children, the formula has been utilized,

(7) $$s_f = \pm \sqrt{\sigma^2 - s^2}.$$

As has been said, the traits utilized in this study are the breadth of the nostrils, the interpupillary distance, the height of the ear, and the length of the middle finger. These were selected arbitrarily, and any of the thirty-odd traits which have been measured in the course of the investigation might have been utilized. The first step was the computation of the averages and mean square deviations of the general population in the four traits selected. The results are given in Table I. The school, it may be remarked, is primarily a boys' school, and the only girls are those in the lower grades. Only enough girls were measured, therefore, to allow of their utilization in the fraternities, and enough others to permit age-means to be computed. As has been said, since the families are so small, it is necessary to work with deviations of the individuals from the averages of their ages. It may be remarked in passing that the figures show what would be expected, where the age-groups are large enough to be of significance—that there is a steady increase in the means, and that the variability increases with the period of most rapid growth. The data on the girls, because of the shortness of the series and its smallness, is irregular. A control group of 31 boys, remeasured to check up observations, gave the following observational errors:

For interpupillary distance	.46 mm.
For width of nose	.48 mm.
For height of ear	.84 mm.
For length of middle finger	1.09 mm.

TABLE I

COLORED CHILDREN, NEW YORK CITY

Age	Sex	*N*	Mean	$\pm\sigma$	*N*	Mean	$\pm\sigma$
		Interpupillary Distance (mm.)			Width of Nostrils (mm.)		
5–6	M	21	54.23	2.56	22	31.46	2.51
6–7	M	52	57.85	3.41	55	32.50	2.37
7–8	M	98	57.03	2.50	102	33.44	2.35
8–9	M	121	58.56	2.89	125	33.67	2.41
9–10	M	126	59.63	2.65	131	34.84	2.91
10–11	M	133	59.61	2.60	136	35.08	2.31
11–12	M	136	61.91	2.92	140	35.79	2.66
12–13	M	124	61.67	3.25	126	36.24	2.98
13–14	M	140	62.52	3.20	141	37.74	2.95
14–15	M	116	63.23	3.21	117	38.84	3.18
15–16	M	80	64.24	3.20	81	40.16	3.06
16–17	M	31	64.87	3.30	30	39.87	3.39
17–18	M	12	63.91	2.69	12	41.50	2.04
18–19	M	2	62.50	....	3	36.60	....
5–6	F	21	54.09	3.28	22	31.96	1.89
6–7	F	14	55.79	2.78	14	32.58	1.84
7–8	F	35	56.83	2.46	36	33.03	2.26
8–9	F	25	57.64	3.12	26	33.43	2.24
9–10	F	6	55.16	....	6	33.50	....
10–11	F	8	58.62	....	8	32.87	....
11–12	F	4	59.25	....	5	35.00	....
12–13	F	1	60.00	....	1	36.00	....
		Height of ear (mm.)			Length of middle finger (mm.)		
5–6	M	22	52.71	4.33	22	70.37	4.65
6–7	M	55	53.48	4.40	55	73.21	4.54
7–8	M	102	53.50	3.59	102	75.64	4.68
8–9	M	125	54.95	3.40	124	78.95	5.05
9–10	M	130	54.90	4.59	130	81.50	4.68
10–11	M	136	55.79	3.90	135	84.27	5.06
11–12	M	140	56.73	4.02	140	88.05	5.74
12–13	M	126	56.90	4.03	126	90.55	6.29
13–14	M	141	56.70	3.79	140	94.45	7.04
14–15	M	116	57.74	3.87	116	98.97	6.28
15–16	M	81	58.36	4.19	81	103.15	6.89
16–17	M	30	57.73	3.24	28	102.14	7.51
17–18	M	12	57.75	3.58	12	104.58	5.17
18–19	M	3	55.00	....	3	98.66	....
5–6	F	22	54.05	4.40	22	70.77	3.45
6–7	F	14	53.07	2.08	14	72.35	4.97
7–8	F	18	52.38	2.81	35	76.12	4.89
8–9	F	26	51.88	4.28	26	77.11	5.37
9–10	F	6	53.33	....	6	78.00	....
10–11	F	8	53.12	....	8	78.87	....
11–12	F	5	54.20	....	5	83.20	....
12–13	F	1	56.00	....	1	98.00	....

These figures are so small, when the mean lengths of the traits measured are considered, that no allowance for them need be made. However, since the correlations were to be carried out in terms of the deviations of the individual members of the fraternities in percentages of the general sigmas represented by these deviations, it was necessary to throw together the age and sex groups, and ascertain the mean square deviation for the entire series, correcting it for skewness due to the factor of growth, the corrected σ being

$$(8) \qquad \sigma \text{ corr.} = \pm \sqrt{\sigma^2 - \frac{\Sigma n d^2}{N}}$$

where σ^2 is the uncorrected mean square deviation of these age and sex groups thrown together, n the number of cases in each sex-group, d the deviation of the mean of that group from the mean of the total series, and N the total number of cases in all age and sex groups. The results are

	Interpupillary distance	Width of nostrils	Height of ear	Length of middle finger
N =.....	1306	1339	1319	1331
Mean =.....	60.32	35.77	55.77	86.43
σcorr =.....	2.94	2.77	3.83	5.62

Following this, it was possible to throw the deviations from the age averages into terms of the respective general sigmas for the various traits.

The coefficients of fraternal correlation were then computed, using the Pearson product-moment formula and working with the percentages which had been calculated. It was necessary, in each instance, to separate those children belonging to fraternities of differing numbers of children, for the reasons explained above. However, there were too few fraternities of four children to allow the computation of correlations of any significance, and therefore the seven fraternities of this number were divided into fourteen pairs of two children each, and correlated along with the other fraternities of two. The differences in numbers which are apparent in all four series in almost every instance are due to numerous causes—in that of interpupillary distance the occurrence of cross-eyed children made it impossible to take this measurement, and with some of the younger children measurements such as nose-width are difficult, and sometimes cannot be obtained at all. Where two members of a fraternity of three had been measured for a trait, and the third child had not been, the two were correlated, for that trait, with the fraternities of two. In the correlations, therefore,

	Interpupillary distance	Width of nose	Height of ear	Length of middle finger
n_2 =.....	380	392	390	392
n_3 =.....	135	153	153	153

and the resulting correlations were

fraternities of 2 (r_2) =...	.34	.39	.45	.24
fraternities of 3 (r_3) =...	.42	.55	.40	.27

With these data, it is possible to proceed directly to the computation of the mean square variability of the family strains, s. In doing this, the final formula (2) was utilized, since the result desired was that for families of an infinite number of children. However, to have an actual check on the accuracy of the work done formula (1) was used in the case of the trait height of the ear, giving the result

$$s_{m_2} = \pm 3.26$$
$$s_{m_3} = \pm 2.95$$

After this result had been derived, the writer then actually averaged the deviations (not thrown into terms of percentages of the general sigma, however) of the individuals in each fraternity, and took the mean square deviation of these averages, the result being

$$s_{m_2} = \pm 3.24$$
$$s_{m_3} = \pm 2.92$$

the difference not being enough to be considered. Therefore, the calculation of s was proceeded to, by formula (2) for an infinite number of children, with the following results:

	Interpupillary distance	Width of nostrils	Height of ear	Length of middle finger
s_2 =.....	±1.71	±1.73	±2.56	±2.75
s_3 =.....	±1.90	±2.05	±2.42	±2.92

And the errors of these, calculating from the formulae (3) and (4) given above, were

ϵ_2 =.....	± .67	± .60	±1.31	±2.45
ϵ_3 =.....	±1.12	±1.06	±1.78	±3.32

These errors are, of course, essential to obtain the final results, which are to be had through combining the variabilities for the families of differing numbers of children. This is done by the usual method of weighting, *i. e.*, multiplying each by the reciprocal of the square of the error, adding the two, and dividing by the sum of these reciprocals, the formula for which may be expressed as

$$S = \frac{\left(s_2^2 \cdot \frac{1}{\epsilon_2^2}\right) + \left(s_3^2 \cdot \frac{1}{\epsilon_3^2}\right)}{\left(\frac{1}{\epsilon_2^2} + \frac{1}{\epsilon_3^2}\right)} \tag{9}$$

This gives a final figure for the variability of the family strains for the Negro-White New York community in these traits which have been

treated, and, at the same time makes it possible to compute directly the variability within the family lines, since the general sigma, squared, should equal the sum of the squares of the variabilities of the family lines plus that within the families, as has been shown in formula (7). Therefore, we have:

	Interpupillary distance	Width of nostrils	Height of ear	Length of middle finger
s =.....	±1.76	±1.81	±2.51	±2.81
s_f =.....	±2.35	±2.10	±2.88	±4.87

It is to be noted that, in every case, the variability of the family strains is smaller than the variability within the families, and in two instances very much smaller. To understand fully the significance of this, it may be well to give here the comparative figures for the various populations for which these variabilities were computed by Boas, for Tennessee Blue Ridge Mountaineer inbred families computed by Gordon,[1] and for the Negro-White population of New York, all for the trait cephalic index. These are:

(Potenza).....	s = ±2.41	s_f = ±2.52
Central Italians.....	±2.39	±2.72
Bohemians.....	±2.37	±2.61
Worcester, Mass.....	±2.36	±2.36
East European Jews.....	±2.29	±2.52
Scotch.....	±2.17	±2.66
New York Negroes.....	±1.85	±2.93
Blue Ridge Mountaineers.....	±1.85	±2.09
Chippewa.....	±1.77	±3.32
Mississauga Chippewa.....	±1.47	±3.10
Bastards.....	±1.26	±2.52

Here we see, that in the comparative series, the New York mixed Negro population stands low in the variability of family strains, while the variability within the families is high. The figure which was worked out for cephalic index, which is given above, while not strictly comparable to that for the traits utilized in this paper, is, at the same time, checked by them, at least in so far as the relative standing of the two variabilities is concerned. That we have such a low figure for the variability of the family strains would seem somewhat surprising, at first glimpse, since it is generally considered that the mixed Negroes of this country are most heterogeneous. That there is large diversity within the families shows this. It is not felt that misinformation as

[1] Gordon, Isabel, *Reduction of Variability in an Inbred Population*. MS (Doctoral Dissertation, Columbia University, 1925).

to whether actual brothers and sisters were really measured has affected the results. The greatest care was taken to see that only actual brothers and sisters were correlated. On this there was a double check—the school records were carefully gone through and then each child, as he was measured, was asked if he had brothers and sisters in the school, and was requested to name them and tell the grades in which they were to be found. Thus, it is felt that only actual relationships are included in the figures.

Is it true, then, that we have here the phenomenon of a relatively homogeneous group of people? It would seem so, and the large figures for the variation within the families would further lead us to the same belief. That there has been a great variety of strains entering into the composition of the American Negro-White population there can be no doubt, for the White ancestry includes individuals from all the world, and certainly from every branch of the White race. At the same time, the crossing has not gone on in the recent past as much as is ordinarily thought. These individuals are all small children. In their parent generation, the writer had occasion to take some 125 genealogies, and it was found that very few individuals were of the F_1 generation of a Negro-White cross, and not very many of the F_2. That would mean that most of these children, if the genealogical sample is at all representative of the general population, would be at least F_3 or farther removed from the original cross. Then we should expect just what we find, that there is a relatively homogeneous group, which is the result of extensive crossing some generations back. And if it be objected that the children represented here come from all parts of the United States and even from the West Indies, the reply must be that to solve this question, larger researches, based upon the characteristics of the African Negro, and more historical and genealogical work, is essential.

A FORM OF SATURATION CURVE[1]

By Lowell J. Reed

It is customary to call certain forms of equilibria saturation points, this terminology being used when we are concerned with the maximum amount of one substance that is absorbed by some other substance. Illustrations of these equilibria are common in all branches of science, as for example, a saturated salt solution, a market saturated with goods of a certain type, or a population saturated with hookworms. In many of these cases we are concerned with the ultimate limit of saturation, but often the method or rate of approach to this position is of even greater importance. What this form of approach may be is dependent on the particular problem in question. For instance, in the case of the saturated salt solution, the quantity of water, its temperature and the type of salt will determine the amount of salt that can be taken up. The saturation rate curve is given by the equation for the catalytic reaction.[2] Similarly in dealing with population growth the autocatalytic curve becomes the saturation rate curve, and expresses the time rate of saturation of a given piece of land with human beings; the limiting position depending among other things on the amount of land, its character and the type of people.

These two cases are illustrations of approach to a fixed level, but many problems arise in which the limit itself is variable, as, for example, the cases where population is the absorbing medium and therefore determines the level of saturation. A particular example of this kind that is attracting considerable attention at the present time occurs in the automobile industry, the point under discussion being the number of automobiles the population can absorb. There were at the end of 1924 enough registered passenger cars in this country to allow one car to every 7.4 persons, and as the present high rate of growth in registered cars cannot continue indefinitely, it is obvious that the growth in automobiles must be considered in connection with the growth in population.

To do this let us review the argument used in connection with the autocatalytic curve.

[1] Papers from the Department of Biometry and Vital Statistics, School of Hygiene and Public Health, Johns Hopkins University. No. 118.

[2] T. B. Robertson, *The Physical Chemistry of the Proteins*, Longmans Green and Co., New York City, 1918, pp. 276–277.

If y equals the population at time x, K equals the limiting value of y, and b_1 equals the inherent rate of growth of y, then the equation

$$\frac{dy}{dx} = -\frac{b_1}{K} y(K-y) \qquad (1)^1$$

expresses the fact that the rate of change of population at any time is proportional to the population at that time and to the difference between the limiting population and the population attained (*i. e.*, to the room remaining for population expansion). This rate equation when integrated leads to the autocatalytic form

$$y = \frac{K}{1+e^{a_1+b_1x}} \qquad (2)$$

in which the new constant a_1 is the constant of integration and is therefore determined by the position of the origin.

If now, z equals the number of automobiles at time x, b_2 equals the rate of increase of automobiles, and m equals the limiting number of automobiles per person then the following equation relates z to its limiting factor, my, by exactly the same assumptions that were used to relate y to its limiting factor k.

$$\frac{dz}{dx} = -\frac{b_2}{my} z(my-z) \qquad (3)$$

Substituting the value of y given in (2) into this equation we have

$$\frac{dz}{dx} = -b_2 z + \frac{b_2(1+e^{a_1+b_1x})}{mK} z^2$$

This equation is one of the standard forms treated in any textbook of differential equations and the solution is given by

$$\frac{1}{z} e^{-b_2x} = c - \frac{b_2}{mK} \int (1+e^{a_1+b_1x}) e^{-b_2x} dx$$

$$= c + \frac{1}{mK} e^{-b_2x} - \frac{b_2}{mK(b_1-b_2)} e^{a_1+(b_1-b_2)x}$$

$$z = \frac{mK}{1+cmKe^{b_2x} + \dfrac{b_2}{b_2-b_1} e^{a_1+b_1x}}$$

[1]The negative sign is used in this equation merely for the sake of having positive signs for all the constants in the final equation.

or

$$z = \frac{mK}{1 + e^{a_2 + b_2 x} + \frac{b_2}{b_2 - b_1} e^{a_1 + b_1 x}} \tag{4}$$

where

$$e^{a_1} = cmK$$

The effect of population growth on automobile growth is evident in the term $e^{a_1 + b_1 x}$. Equation (4) is the growth equation for any population that is dependent on or limited by a population that is following the autocatalytic equation.

Let us now apply this theory to numerical observations regarding automobiles[1] and population in the United States.

Taking the origin at January 1, 1910, the equation for the United States population[2] is

$$y = \frac{197.274}{1 + e^{.13535 - .03134x}} \tag{5}$$

Using the values of K, a_1, and b_1 as given in this equation in the general equation (4) and determining the constants m, a_2 and b_2 to fit the number of registered passenger cars in the United States, we have the following equation to express the growth in automobiles.

$$z = \frac{38.636}{1 + e^{4.8203 - .3450x} + 1.1e^{.13535 - .03134x}} \tag{6}$$

The calculated values from equation (6) are compared with the observed values in Table I and in Figure 1. The fit is evidently very satisfactory.

To bring out more clearly the connection between this curve and the population curve we may plot them on arithlog paper. See Figure 2.

This graph shows clearly the very great rate of growth of registered passenger automobiles in the period 1905 to 1920, with the beginning of a bend in the curve evident after 1920. The later part of the automobile curve is parallel to the population curve and shows that the automobile curve takes on the same rate of growth as the population curve as saturation begins to take effect. The decade 1925 to 1935 is evidently a critical period.

[1] For the data on which this illustration is based I am indebted to Mr. George R. Wadsworth. His figures were compiled from the records of the National Automobile Chamber of Commerce, United States Bureau of Public Roads and "Automotive Industry."

[2] R. Pearl and L. J. Reed, "The Rate of Growth of the Population of United States Since 1790 and Its Mathematical Representation," *Proceedings National Academy of Sciences*, 1920. Vol. 6, pp. 275–288.

TABLE I

COMPARISON OF REGISTERED PASSENGER CARS AND POPULATION IN UNITED STATES

Date January 1	Registered passenger cars,[1] (000 omitted)		Population, (000 omitted)	Persons per car	
	Observed	Calculated		Observed	Calculated
1900	8	10	76,870	9,609.	7,687.
1901	12	14	78,344	6,529.	5,596.
1902	19	20	79,830	4,202.	3,991.
1903	27	28	81,323	3,012.	2,904.
1904	35	39	82,826	2,366.	2,124.
1905	43	55	84,335	1,961.	1,533.
1906	62	78	85,852	1,385.	1,101.
1907	88	110	87,375	993.	794.
1908	121	155	88,903	735.	574.
1909	171	218	90,436	529.	415.
1910	272	306	91,972	338.	306.
1911	412	429	93,513	227.	218.
1912	554	600	95,056	172.	158.
1913	825	836	96,600	117.	116.
1914	1,160	1,160	98,146	84.6	84.6
1915	1,540	1,598	99,691	64.7	62.4
1916	2,130	2,184	101,237	47.5	46.4
1917	3,310	2,951	102,781	31.1	34.8
1918	4,670	3,931	104,322	22.3	26.5
1919	5,620	5,146	105,861	18.8	20.6
1920	6,809	6,597	107,394	15.8	16.3
1921	8,221	8,255	108,928	13.3	13.2
1922	9,321	10,064	110,454	11.9	11.0
1923	10,860	11,938	111,975	10.3	9.38
1924	13,465	13,786	113,489	8.43	8.23
1925	15,526	15,526	114,996	7.41	7.41
1926	....	17,099	116,495	....	6.81
1927	....	18,477	117,985	....	6.39
1928	....	19,657	119,467	....	6.08
1929	....	20,653	120,939	....	5.85
1930	....	21,490	122,397	....	5.70
1935	....	24,184	129,530	....	5.36
1940	....	25,829	136,318	....	5.28
1945	....	27,185	142,714	....	5.25
1950	....	28,416	148,678	....	5.23

[1] The registration given in this table as of January 1 of each year is the actual registration of December 31 of the previous year, but is used as of January 1 to set against the population figures of that date.

We may bring out the relationship between these two curves in another way by considering the ratio of persons to automobiles. Since we wish to do this for each year and the population is counted only once in ten years, it is necessary to interpolate the population figures. For these interpolated figures I have used the results of equation (5). In no case do these values differ from those obtained by arithmetic interpolation between census values by as much as two per cent. Since the number of cars applies to December 31 of the year in question I have used the population of January 1 of the following year to set against this number of cars. Table I shows the actual and theoretical number of persons per car for each year and these values for the years 1913 to 1924 are plotted in Figure 3, this shorter period being used in the graph to bring out more clearly the approach of the curve to its asymptote.

The limiting value, 5.1 persons per registered automobile, is the reciprocal of the constant m of the theoretical discussion and is found by taking the ratio of the numerators of equations (5) and (6).

These three comparisons show clearly that the theoretical equation developed does well represent the facts as to growth in registered automobiles and that it also gives a logical relationship between automobile growth and population increase. While the theory agrees satisfactorily with observed facts it is evident that it does not cover the

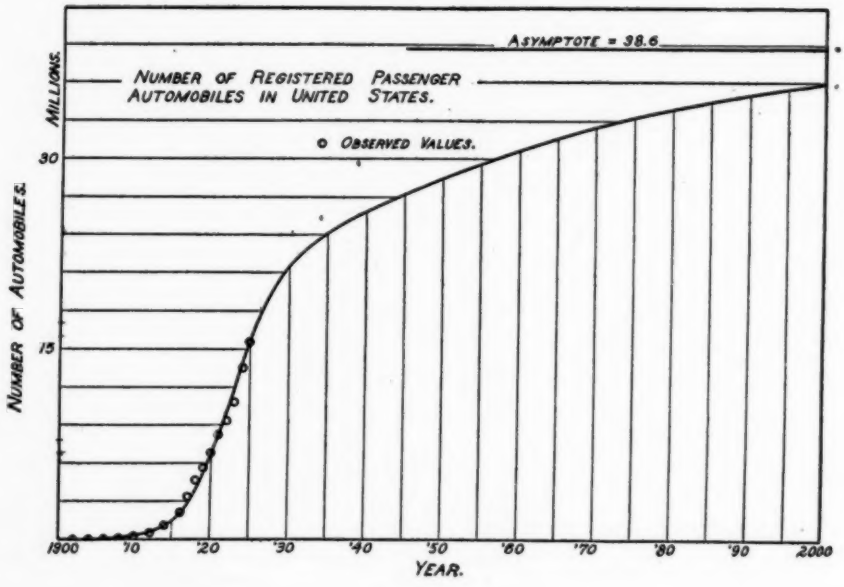


FIGURE 1

possibility of the introduction of some other method of transportation that might supplant the automobile causing it to go the way of the bicycle and the horse and buggy.

Many other applications of this theory can be found in the field of economics, but the most interesting ones are perhaps in the field of biology. Here the variable z becomes a biological population that is dependent on the human population. The course of development of any domesticated plant or animal, newly introduced into a human population, would be expected to follow the curve given by the general equation of this theory.

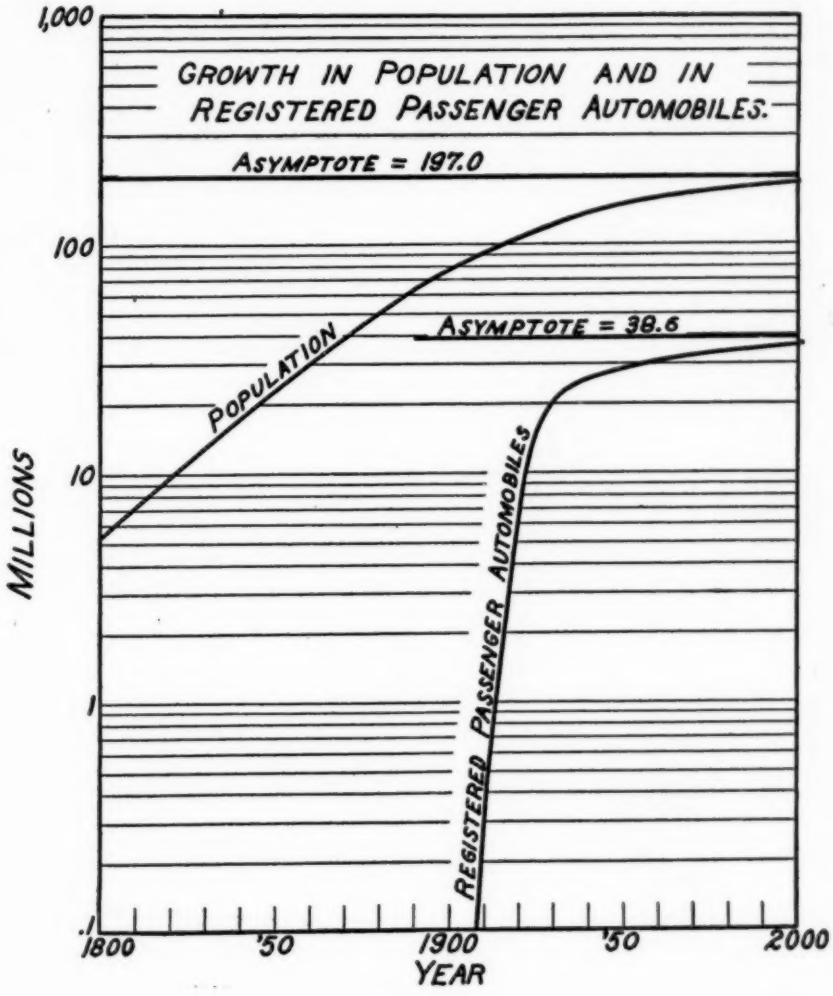

GROWTH IN POPULATION AND IN
REGISTERED PASSENGER AUTOMOBILES.
ASYMPTOTE = 197.0
ASYMPTOTE = 38.6
POPULATION
REGISTERED PASSENGER AUTOMOBILES
MILLIONS
1,000
100
10
1
.1
1800
'50
1900
'50
2000
YEAR

FIGURE 2

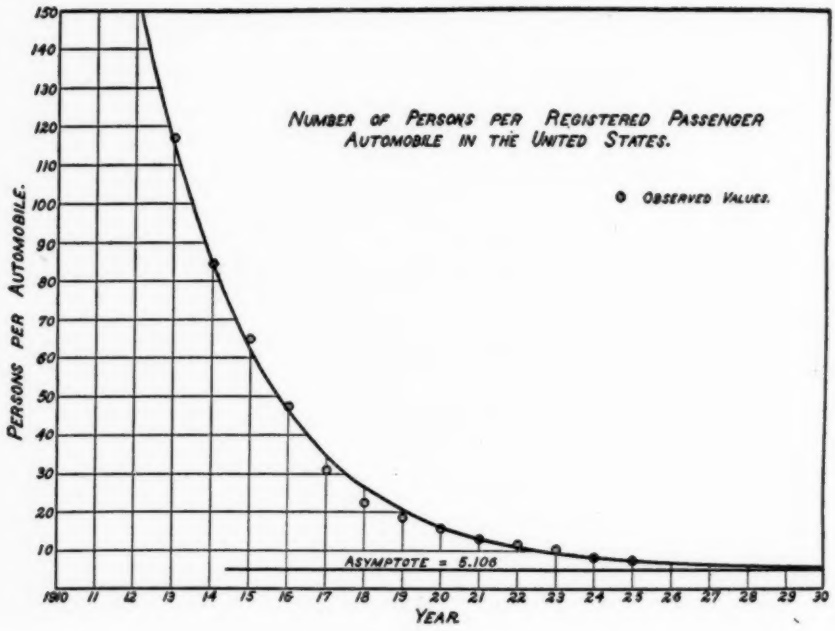

NUMBER OF PERSONS PER REGISTERED PASSENGER AUTOMOBILE IN THE UNITED STATES.
OBSERVED VALUES.
ASYMPTOTE = 5.106
PERSONS PER AUTOMOBILE.
YEAR.
150
140
130
120
110
100
90
80
70
60
50
40
30
20
10
1910 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30

FIGURE 3

NOTES

THE REVISED INDEX OF THE VOLUME OF TRADE

By Carl Snyder, *Federal Reserve Bank of New York*

In the December, 1923, issue of this Journal report was given of a new and widely inclusive Index of the Volume of Trade, computed by months from the beginning of 1919. This was an attempt to make use, in a composite index, of the store of newer material which had become available in and since the War. It included the new data as to wholesale and retail trade compiled by the Federal Reserve Board, representing a field for which any kind of accurate information had hitherto been lacking; a comprehensive sampling of some thirty different lines of production; the new reports of car loadings divided as to character of traffic; a number of new types of business data, as life insurance, advertising, electric power production, grain and cotton future sales, etc.; and, in addition to all these, an endeavor to restore to a strictly comparable basis a number of important series, given in dollar values, whose meaning and comparability had been completely lost by the unparalleled rise and fall of prices in and after the War.

In all fifty-six separately computed series were included; each, with a few exceptions, in terms of percentage variation from the computed line of normal rate of growth or trend of past years. In a few unimportant cases this trend was so difficult to ascertain from the data available that use was made of variations from simple long-term averages.

These fifty-six series were weighted according to the evidence available as to their relative importance or significance as indicators or "barometers" of business, and the weights assigned discussed with a number of statisticians who had worked most extensively in this field. The difficulties and obstacles to this undertaking were set forth in detail in the article cited, and in presenting the findings it was stated that:

> The results here shown are presented simply as first approximations and nothing more. The purpose is to revise the index for all of the series each year in the light of such new information as may be available as to seasonals, the computation of normal growth, price changes, and the like.

Working on this new index as a base or standard of comparison, it was found possible to devise other new measures of trade variations, one from an unexpected source. In the following issue of this Journal

(March, 1924) I gave report of another type of Index of Business Activity derived from the variations in the rate of turnover of demand deposits in the Reporting Member Banks of the Federal Reserve System, detailing the quite remarkable likeness which had been disclosed by comparison of this new index with our weighted composite of the fifty-six series. The resemblance, especially as to the percentages of variation, was striking.

Then, in the September issue, report was given of a third index of business activity derived from the variations of bank debits outside New York City, corrected for the computed changes in the average of all payments or, as it may be termed, the General Price Level, as contrasted with special price indexes, such as for commodities at wholesale, wages, cost of living, etc. In a previous paper[1] it was shown that, by combining a series of these indexes into a weighted composite, giving a relatively low weight for commodity prices at wholesale hitherto regarded as the best index of the general price level, it was possible to derive from corrected bank debits an index of trade so closely similar to the composite Index of the Volume of Trade as to be nearly identical, both in time and in percentages of variation.

The index so constructed from debits stood the further test of continuing the line of growth of bank clearings for the forty years preceding the War, straight through and after the War without interruption, precisely as was possible in the case of a number of quantitative series, such as pig iron production, railway traffic in ton miles, etc.

It was further found that other of the separate series included, as, for example, electric power production in kilowatt hours, likewise yielded quantitative indexes closely similar both in percentages and in time relations to the composite index. All of which, in its cumulative effect, tended to give us a degree of confidence in the original results such as, of course, no one could have had in the beginning; and the clearing up of an unexpected difficulty was to carry this confidence still further.

During the last year and a half we have carried through the original plan of revision of each of the component series, and, in addition, this period, of further return towards more normal conditions, has itself supplied new evidence that the major industrial trends were not materially changed by the War or by the post-War boom and collapse; and that the foundations of these investigations are solid. It was possible, also, to obtain considerable new data in some fields where the available series were of recent origin and the computation of the secular trend or rate of growth was extremely difficult. The results of this extended revision are presented herewith.

[1] "A New Index of the General Price Level," this JOURNAL, June, 1924, p. 189.

THE PROBLEM OF FIVE SUNDAYS

It is of interest that, aside from two or three series in which it was possible to compute the trend of growth more accurately, the most important revisions were those which arose from the chronic problem of five Sundays, that is, the consequences growing out of an extra Sunday in the month, reducing the number of business days by, on the average, about $\frac{1}{25}$. It had seemed to us in the beginning that for this purely calendar effect there would be a compensatory movement, *i. e.*, that more business would be done the day before or the day following, and that therefore the calculation of the normal seasonal allowance would not be greatly disturbed. But it soon became clear that this was not true and that, as series after series show, the data were far more sensitive than could have been anticipated. The evidence piled up that one less business day in the month meant nearly that much less trade in that month, not merely in specially selected lines but in almost all the lines investigated; and therefore that this effect was accurately registered in the composite index of the whole fifty-six series.

To take a concrete example: a year ago, in the summer of 1924, there was serious depression in a number of industries, and, according to our index, a drop of general business to a shade below the computed normal. September showed a marked improvement and this was continued in October and likewise in December and the months following. But November showed a sharp break in the line, amounting to four points and over. It was clear that in November there had been no real interruption of the steady trend of improvement and that the drop was due to the calendar effect of five Sundays falling in a short month with two holidays. This was substantiated by the recalculation of a number of important series on a daily basis.

This five-Sunday effect had been repeated so often and with results so nearly similar that there could be no question of the source of the difficulty. In consequence, nine of the most important series, making up by weight approximately one-half the composite index, have been recalculated throughout on a daily basis, and it is to these recalculations that the major changes shown in the revised composite are due. With a single exception, in no case did this revision amount to more than four points, and the major part of the changes have been due to this purely calendar effect.

It is obvious that, unless the data of the constituent series were not themselves accurate to a surprising degree, this result would have been improbable.

GROUP CHANGES

Among the five major groups into which the series were divided, the only notable change was that relating to retail trade. Here it had been possible to obtain data, especially in relation to department store sales, from a number of important stores, by years, some for twenty years or more. From these and the later experience it was evident that the gradient shown in the four years and a half for which the new Federal Reserve Board computations had been available was slightly steeper than that of the more extended data. The effect of this change was to make evident that the computations for the last two years or so, as to department store sales and chain store sales, were several points too low. The result, together with a recomputation of these series on a daily basis, was to make the line of retail distribution even smoother than before; and to make clear how little, as a rule, the volume of retail trade varies from month to month or year to year, save in its normal seasonal change and secular growth.

In the important group representing general business activity, which includes bank debits corrected for the computed changes in the averages of all payments, postal receipts, electrical power production and long distance communication, the changes were chiefly due to recomputation on a daily basis. The general steadiness of this line (almost a replica of the composite index of the whole fifty-six series) as compared with the erratic movements of the group representing financial activity and speculation, reveals how little the latter is associated with real changes in business. In the latest six months of this computation the index of this activity has risen to levels that, relatively, have been reached only twice before in the last half century, while the volume of general business and the total volume of trade, although high, has not exceeded the limits reached in other prosperous periods in the last twenty-five years or more.

Among the producers' goods series, silk consumption was substituted for wood pulp, no longer available, and the rail equipment series became simply locomotives built, car construction being no longer available. The internal revenue receipts from the theatre tax being no longer comparable, this series was dropped.

AN INDEX OF REAL ESTATE ACTIVITY

By courtesy of the National Association of Real Estate Boards it has been possible to add an interesting new series measuring activity in urban real estate. This Association has collated the number of deeds recorded in forty-one representative cities of the country, by months

from 1916; and in addition to this a smaller but excellent sampling of a number of cities from 1900. From these we have calculated the percentage deviations from the normal rate of growth in the usual way; and these deviations, it has been found, bear an interesting general likeness, as might be expected, to building activity, though the rate of growth for the two series is strikingly different. The gradient of increase in the number of real estate transfers in the last quarter of a century averages about 7 per cent per annum, which is more than twice the urban rate of growth in population in this period, and in the face of the fact that the computed rate of growth in the volume of new building has been rather less than 2 per cent per annum.

This is a development rather hard to explain. Apparently speculation in urban property and building in this period has been increasing at a very rapid rate and far beyond the increase in the number of dwellings and buildings.

THE INDEX OF EMPLOYMENT

In the weighting of the several series there has been no change, and in regard to only one has the writer had serious doubts as to its value as an indicator of business conditions. That is the index of factory employment. The returns here used are those for New York State only, but, as Dr. Berridge has shown, these vary singularly little from the combined returns from all the states reporting, or even from those of individual states like Wisconsin or Illinois.

In New York State the returns from the 1,600 reporting factories and corporations showed, in the highly prosperous year of 1925, only about the same number of employees as those reported in 1914. Meanwhile, in the 1920 period this number was greater by 25 per cent, and again in 1923, at the peak, by 16 per cent. As factory production this year has pretty certainly been up to, if it has not exceeded, the average for 1923, and must now be considerably above 1920, we are forced to the conclusion that the man-product, or so-called efficiency of labor, must be considerably higher now than just following the War. All this makes the use of these and similar employment figures as indexes of business conditions extremely difficult. Apparently only the comparison of closely adjacent months is of significance. Inasmuch as it seems probable that, in the 1919–1920 boom, the influence of war conditions was still strong and the high rate of employment then abnormal, and the subsequent trend therefore obscure, the series has been recomputed in the terms of a four-year average from 1921 to 1924 inclusive, as more representative of normal peace-time conditions.

THE MEASURE OF GENERAL TRADE

In the light of this revision of the index and a comparison of the composite with some of its more important constituents, it would seem that we now possess a true measure of the general trade of the country. Obviously such a measure should approach as nearly as possible a complete record of all transactions, in the exchange of goods and services, either in quantities or at a given base of prices. Complete or astronomical knowledge would be the total of every transaction of barter or sale in the country.

In the total of bank debits, or amount of checks drawn, we have a close approach towards such knowledge, at least so long as the average of all these payments does not notably vary. The amount of total transactions done by checks has been estimated at around 85 per cent of the total exchanges of the country.

If we could estimate accurately the changes in the average of all kinds of prices, retail, wholesale, wages and all the rest, then from the reported amount of debits or checks drawn we should have a pretty accurate measure of general trade. This is precisely what we have attempted to do by means of our debits index. In its secular rate of growth the volume of bank debits so derived corresponds closely in recent years with the computed rate of increase of industrial production, and in its oscillations above and below this normal line of growth it likewise corresponds very closely, as already indicated, with the composite Index of the Volume of Trade.

Parallel results have been derived, as noted, from the variation of bank deposit activity and, it may be added, from such an unexpected source as the stock market. An index of industrial stocks derived from the variations from the trend, properly damped, yields a strikingly similar picture; which seems to negate the idea that the stock market is a reliable forecaster of business. It is sometimes a leader, sometimes a laggard, but more often than either a close companion.

It seems scarcely possible that results so generally consistent could be obtainable if they did not approach pretty closely to the reality of things. In the mind of the writer, therefore, after a rather long travail, it seems possible now to measure the nation's trade, from month to month, with a certainty and an accuracy that hardly anyone could have dreamed of five years ago. Having at last a standard of comparison, it is now equally possible to consider the reliability of various other measures which have at times been employed.

The most striking thing that has been revealed by all these investigations is the general synchronism of the greater part of the included series. There are some laggards among which, as a rule, is iron produc-

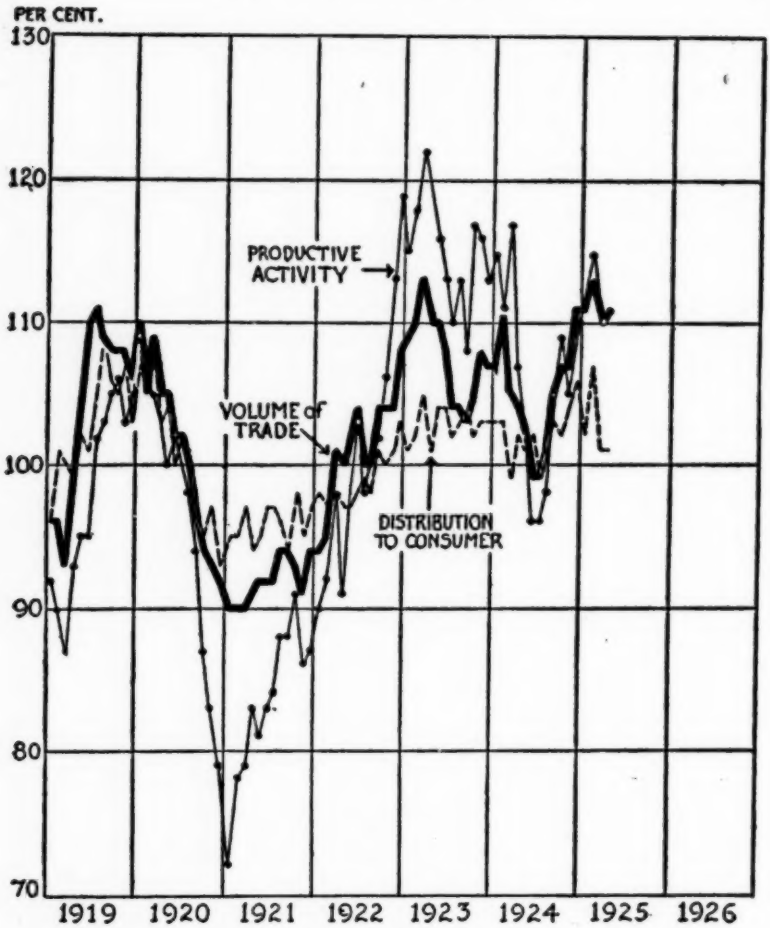
tion, so that any index heavily weighted with iron production can scarcely be regarded as a reliable index of general trade. And for the same reasons, this is true of railway traffic, or total car loadings.

In general, production in basic industries and the distribution of these products, which are so heavy a part of the traffic of our railways, varies far more widely, as might be expected, than the volume of the nation's exchanges, so large a part of which is retail trade or final consumption and, scarcely less, the exchange of human labor.

On the other hand, there have been found, it is regrettable to say, no constant leaders. The problem of forecasting the broad movement of trade seems as difficult as ever, though some interesting general results have been attained. We seem able now pretty accurately to mark the limits, so far as the actual variations in national trade are concerned, of expansion or decline in the most vigorous of booms or the severest of depressions. It is at least some gain to know that when business is at its worst, in popular parlance "dead," it is still proceeding at about 90 per cent of its normal rate of growth.

The full tables of the revised series follow.[1]

[1] This revision, under Miss Alice Carlson, has been chiefly the work of Lucile Bagwell, Dorothy Thomas, F. C. Knote, Nancy McLeod, and H. V. Roelse.



Showing the revised index of the total Volume of Trade, with computations on a daily basis, compared with the revised index of retail trade and of productive activity (which includes building construction, etc.). See this JOURNAL, Dec., 1923, p. 952.

INDEXES OF THE VOLUME OF TRADE

	Total volume of trade	Productive activity Group weight, 25%						Primary distribution Group weight, 22%									
		Produc-ers' goods	Consum-ers' goods	Employ-ment	Motor vehicles	Building permits	Group averages	Miss. car loadings	Other car loadings	Wholesale trade	Exports	Imports	Cereal exports	Panama Canal	Group averages	Dept. stores	Ch st
Weights	100	9	8	6	2	4		5	2	8	3	2	1	1		8	
1919																	
Jan.	96	103	106	112	56	24	92	101	96	94	118	61	142	86	98	93	
Feb.	96	97	97	111	69	36	90	101	89	92	131	70	110	95	98	104	
Mar.	93	90	95	109	73	41	87	97	83	89	130	74	148	80	92	99	
Apr.	100	90	107	110	82	54	93	90	110	105	160	81	227	77	112	100	
May	105	86	106	111	97	67	95	96	106	113	138	100	248	90	116	100	
June	110	89	97	110	97	80	95	95	106	120	222	92	256	92	129	101	
July	111	98	101	115	124	81	102	101	113	127	146	107	104	85	118	102	
Aug.	109	102	101	117	96	93	103	104	102	110	151	87	109	108	111	110	
Sept.	108	99	112	117	102	86	105	106	112	107	116	126	113	97	110	104	
Oct.	108	97	110	114	133	95	106	103	105	102	106	109	81	92	107	99	
Nov.	108	89	103	115	139	96	103	110	87	109	122	115	89	79	107	103	
Dec.	106	96	109	119	142	99	105	111	103	124	108	98	77	126	113	100	
1920																	
Jan.	110	106	113	122	111	86	109	112	106	115	120	118	74	114	113	106	
Feb.	105	109	103	121	123	71	106	107	102	93	122	118	74	108	103	100	
Mar.	109	108	104	123	127	62	105	109	108	99	144	118	140	110	112	106	
Apr.	105	103	99	123	119	70	103	93	109	93	123	118	125	110	103	89	
May	105	102	101	122	117	49	100	103	111	104	137	105	224	114	115	104	
June	102	105	103	122	110	56	102	105	119	102	130	140	145	109	112	95	
July	102	99	101	123	121	54	100	105	115	96	138	137	178	110	114	97	
Aug.	100	100	98	120	111	56	98	104	114	88	116	125	142	130	106	95	
Sept.	96	100	92	117	92	51	94	102	112	89	105	93	125	127	100	89	
Oct.	94	95	84	114	58	52	87	101	110	79	117	84	139	107	96	89	
Nov.	93	90	84	106	48	46	83	99	115	79	110	86	116	112	95	96	
Dec.	92	85	84	98	34	51	79	97	114	80	124	75	139	121	97	93	
1921																	
Jan.	90	72	81	92	26	48	72	89	93	84	130	63	164	118	95	102	
Feb.	90	71	87	94	39	68	78	92	87	88	121	72	163	109	96	103	
Mar.	90	68	91	94	65	65	79	90	79	93	96	80	189	115	96	104	
Apr.	91	67	96	93	85	78	83	91	90	94	92	92	214	95	98	100	
May	92	68	87	92	90	79	81	91	92	100	96	79	246	76	100	103	
June	92	68	91	90	83	88	83	91	90	101	106	77	232	75	101	104	
July	92	63	94	90	94	94	84	91	86	99	114	76	194	73	99	106	
Aug.	94	68	101	90	89	105	88	91	83	104	121	78	305	87	107	98	
Sept.	94	69	97	92	81	108	88	94	84	98	91	75	201	78	97	94	
Oct.	93	73	96	93	77	123	91	98	89	101	82	72	109	89	94	103	
Nov.	91	67	91	92	71	115	86	92	82	101	70	82	80	81	89	99	
Dec.	94	75	92	92	48	117	87	92	82	99	69	90	94	89	90	101	
1922																	
Jan.	94	77	93	91	49	131	90	92	89	97	70	83	139	70	91	99	
Feb.	95	80	97	94	61	124	92	94	100	96	74	85	157	80	94	101	
Mar.	101	85	102	94	70	142	98	96	102	98	92	92	206	85	101	101	
Apr.	100	78	87	94	86	126	91	100	72	101	91	83	201	91	99	100	1
May	102	85	93	96	102	137	98	103	78	99	90	99	198	93	101	99	
June	104	87	95	98	125	151	103	105	78	101	102	105	205	88	104	99	
July	100	84	95	99	123	123	98	103	79	96	98	99	171	104	100	102	1
Aug.	100	85	100	101	124	135	102	98	80	102	90	102	203	100	102	100	
Sept.	104	90	102	102	97	132	102	97	94	106	79	111	183	98	103	103	1
Oct.	104	97	101	104	114	134	106	97	95	111	79	96	121	108	101	103	
Nov.	104	103	110	105	132	146	113	102	108	112	81	102	101	112	104	101	
Dec.	108	106	108	107	123	183	119	103	113	112	71	100	86	119	102	103	1
1923																	
Jan.	109	108	107	108	115	158	115	106	108	109	74	110	96	115	103	98	9
Feb.	110	107	107	109	122	173	118	105	104	113	79	105	129	123	106	98	9
Mar.	113	111	112	110	133	181	122	105	110	110	83	125	106	142	108	103	10
Apr.	110	112	109	111	138	154	119	109	124	107	82	122	107	157	109	97	10
May	110	115	109	112	146	124	116	109	120	103	83	130	143	150	110	100	9
June	108	110	103	111	152	123	113	105	119	103	90	119	92	156	106	102	10
July	104	105	99	111	151	118	110	104	119	105	95	110	73	166	106	98	9
Aug.	104	106	103	110	145	135	113	105	117	107	92	99	86	154	106	104	9
Sept.	103	103	93	109	140	130	108	103	107	104	95	94	86	155	103	104	10
Oct.	105	103	104	110	159	167	117	104	101	109	85	106	58	132	102	100	9
Nov.	108	102	101	107	161	172	116	106	106	100	83	100	42	144	99	101	10
Dec.	107	96	100	105	151	173	113	103	102	97	88	97	53	160	98	102	10
1924																	
Jan.	107	102	104	105	142	170	115	104	105	103	88	100	62	146	101	100	9
Feb.	111	109	110	106	151	207	125	108	110	103	95	117	64	147	105	101	9
Mar.	105	99	102	106	131	197	117	104	103	97	84	102	73	138	99	98	10
Apr.	104	94	104	103	125	141	107	104	102	100	90	112	75	129	101	97	9
May	102	84	103	100	108	141	102	101	101	91	91	110	65	129	96	98	9
June	99	75	99	98	92	139	96	96	95	88	89	105	74	125	94	100	9
July	99	74	104	95	113	122	96	98	92	91	89	108	42	123	93	97	9
Aug.	101	78	110	95	110	137	98	98	93	96	98	92	84	116	96	101	10
Sept.	105	87	110	98	118	142	105	102	100	105	106	104	86	125	104	102	9
Oct.	107	92	103	98	120	147	109	103	98	99	111	104	201	103	107	98	9
Nov.	107	92	99	96	110	155	105	105	103	95	102	101	119	106	101	101	10
Dec.	111	102	103	97	97	171	110	102	108	99	89	108	93	120	101	100	10
1925																	
Jan.	111	107	108	98	100	159	112	104	107	94	94	110	67	95	98	96	9
Feb.	113	105	104	100	110	181	115	108	101	100	90	109	65	100	100	105	9
Mar.	110	101	100	99	119	163	110	105	94	95	103	113	104	106	101	97	9
Apr.	111	100	104	99	134	192	116	106	108	93	95	111	146	97	102	93	9

*Note: One of the series included in this group (weight, 1%) not available for publication.

Distribution to consumer Group weight, 26%								Financial Group weight, 6%					General business Group weight, 17%				
Dept. stores	Chain stores	Chain grocries	Mail order	Life insurance	Real Estate transfers	Advertising	Group averages	New securities	Stock sales	Grain sales	Cotton sales	Group averages	Debits outside N.Y.C.	Debits N.Y.C.	Postal receipts	Electrical power	Group averages*
8	3	6	3	2	2	2		2	2	1	1		8	5	1	2	
93	96	109	98	92	70	91	96	109	71	124	107	99	121	98	104	104	100
104	100	109	90	114	79	97	101	93	73	132	112	96	97	95	102	101	97
99	105	105	81	119	84	101	100	43	127	120	105	94	97	94	97	99	96
100	95	99	91	116	92	106	99	24	170	91	112	99	95	97	105	99	97
100	100	100	97	119	95	110	102	73	204	95	153	134	103	110	106	100	105
101	98	102	91	102	97	119	101	136	195	77	155	149	111	116	98	98	109
102	98	103	105	117	109	122	105	130	204	75	140	147	111	120	100	101	111
110	104	104	114	116	105	123	109	115	145	91	98	118	113	122	94	100	112
104	98	102	110	100	116	124	106	118	143	93	98	119	109	116	98	100	109
99	96	105	118	95	121	123	105	144	121	57	116	110	107	115	101	103	108
103	108	103	117	98	118	125	107	107	178	68	111	125	113	121	93	105	112
100	99	95	116	87	125	123	103	83	147	9	99	95	107	109	101	110	107
106	104	100	114	99	119	127	107	162	117	53	97	118	111	104	102	107	106
100	101	102	117	106	116	135	108	96	129	70	111	105	106	98	101	107	104
106	102	101	101	120	121	129	108	130	171	48	129	133	105	100	108	108	104
89	103	110	85	120	123	124	103	142	168	78	145	141	102	98	114	108	102
104	107	103	88	99	110	127	104	152	98	76	98	112	102	91	103	105	99
95	104	102	83	101	109	129	100	119	55	93	83	87	100	86	104	106	97
97	109	102	90	104	106	129	102	82	74	96	81	82	100	89	108	107	98
95	107	101	92	96	97	134	101	53	81	102	96	78	99	93	105	107	99
89	104	96	79	95	103	127	96	48	90	107	93	79	99	91	104	105	98
89	106	96	71	95	98	126	95	128	86	126	92	106	97	93	102	103	97
96	104	94	89	93	91	122	97	63	130	122	85	99	96	99	101	100	98
93	99	94	76	81	91	117	93	96	142	158	93	121	94	103	101	99	98
102	102	94	72	95	82	109	95	122	95	139	106	113	93	103	96	98	96
103	105	92	73	96	86	103	95	98	60	173	105	99	88	92	98	99	96
106	100	91	85	96	86	100	97	58	96	138	91	98	88	90	99	90	96
100	105	92	74	97	88	94	94	70	91	140	78	90	87	91	97	89	90
103	101	95	73	98	84	99	95	81	101	144	63	95	91	98	94	88	93
104	100	100	76	99	85	98	97	38	107	136	86	86	91	97	98	88	93
106	103	100	73	93	87	98	97	85	54	159	57	82	92	99	95	86	94
98	99	101	78	97	89	95	96	31	65	138	87	70	93	97	102	91	97
94	98	99	81	95	91	95	94	104	77	133	142	105	95	102	98	98	98
103	104	105	75	94	87	94	93	30	77	154	97	78	95	98	93	98	95
99	98	104	70	93	90	96	95	69	90	132	86	96	97	101	97	90	97
101	98	107	71	92	94	95	97	129	100	136	81	113	96	105	100	92	98
99	97	110	77	98	89	94	98	108	96	105	84	100	97	107	94	89	99
101	99	104	73	100	89	92	97	71	94	85	94	85	99	107	97	90	100
101	96	106	79	98	92	93	98	133	133	144	71	125	100	110	98	93	102
100	100	104	80	95	87	94	97	145	178	147	67	143	101	121	97	91	105
99	96	98	85	101	97	95	97	156	168	107	111	144	101	116	100	95	104
99	97	102	84	103	99	98	98	142	142	110	143	136	105	114	103	94	104
102	101	97	88	102	100	99	99	100	88	104	118	100	103	117	99	96	104
100	96	102	81	104	103	99	98	53	104	86	98	83	100	109	105	100	103
103	101	104	85	105	104	96	101	158	126	99	104	129	102	113	102	100	105
103	98	99	93	103	104	101	100	105	149	71	93	113	105	115	99	101	107
101	99	100	93	107	107	101	101	57	112	82	118	90	100	100	102	101	100
103	103	103	100	102	108	101	103	89	114	90	127	104	107	102	102	103	104
8	98	102	98	105	114	100	101	151	115	106	150	145	108	108	104	103	107
8	98	105	100	107	111	101	102	111	133	92	166	124	110	112	102	104	109
3	103	106	103	111	112	102	105	127	150	69	149	132	107	109	105	106	107
7	100	99	103	109	110	104	101	123	116	53	185	119	108	109	99	105	107
0	99	100	113	113	119	103	104	86	134	106	175	120	108	103	104	107	106
2	103	99	101	113	121	102	104	124	114	96	140	119	108	103	102	105	106
8	99	96	107	112	119	104	102	57	72	90	111	77	103	100	99	105	102
4	98	97	98	118	125	101	103	57	76	62	99	71	99	93	103	104	98
4	103	102	103	114	112	100	104	61	84	64	128	80	101	99	97	104	101
2	97	95	104	113	123	103	102	99	91	98	116	91	99	100	101	107	100
1	100	98	99	114	118	104	103	166	130	72	165	138	100	101	102	103	101
7	107	97	105	103	118	100	103	123	147	71	178	132	103	104	102	103	103
0	97	99	106	111	123	98	103	131	154	51	147	128	100	104	101	105	102
9	99	97	107	112	127	102	103	114	119	51	173	115	103	108	104	108	105
1	102	91	98	116	112	97	99	114	105	39	150	105	104	107	96	105	104
2	97	99	108	120	112	101	102	118	104	49	150	107	102	105	102	104	103
8	98	102	105	108	106	99	101	113	77	41	103	111	101	107	101	102	103
8	99	101	109	108	105	98	102	136	97	32	90	98	103	110	94	99	104
4	97	98	95	118	109	93	99	120	139	118	136	129	100	109	99	99	102
2	100	102	105	105	105	92	101	123	125	92	112	117	104	119	96	100	107
1	95	101	114	109	105	97	103	134	104	133	125	122	101	113	100	102	104
1	95	103	110	110	111	97	102	179	165	127	108	134	103	103	101	105	103
1	104	104	111	107	103	97	104	104	238	137	95	153	104	117	94	100	107
1	100	103	125	117	114	97	106	160	246	152	104	178	104	116	103	107	108
1	96	106	114	107	108	93	102	218	237	142	83	189	111	122	98	106	112
1	99	104	120	116	112	97	107	216	187	186	106	183	109	117	98	104	110
1	96	101	107	112	111	97	101	151	218	190	127	176	110	121	95	105	111
1	93	125	109	117	116	96	101	207	141	211	104	169	106	113	103	106	108

THE ASSUMPTIONS IMPLIED IN THE MULTIPLE REGRESSION EQUATION

By Mordecai Ezekiel, *Bureau of Agricultural Economics, United States Department of Agriculture*

In simple correlation we are concerned with the relation between two variables, and ordinarily do not stop to consider whether the relationship is such that one factor varies absolutely or proportionately with the other, though as Dr. Mills has pointed out, it makes quite a difference in the significance of the probable error whether we do or not.[1] When multiple correlation is used, however, clear analysis of the type of the relationship is vital if the very form of the equation is not to imply assumptions we have no intention of making.

In the simplest case, a multiple regression equation of the form

$$X_1 = b_2X_2 + b_3X_3 + \ldots + b_nX_n \qquad \text{(Eq. 1)}$$

assumes that the variation of the dependent variable may be accounted for as the *sum* of the effects of the several independent variables. That is, the net variation in X_1 which accompanies a given value of X_2 is quite independent of the accompanying values of X_3, X_4, etc.

There are many problems in economic analysis where this type of equation may be quite satisfactory. The value of a single product, like hogs, is closely related to the sum of the values of the different products, such as lard, hams, loins, and bacon, which can be made from it; and if the price of hogs were to be correlated with the prices of these different products, the additive form of equation would be quite correct. This statement of the relationships is useful in any case of this general type, where the dependent variable may be considered as merely the sum of several different independent forces.

In many other cases—perhaps in the majority of economic problems—the relations are not nearly so simple. If industrial employment increases by 10 points on the index, will the price of a given goods be expected to react to just the same extent, whether its price is already high or low? If the price of a competing commodity increases from \$5.00 to \$5.50, will the effect on the demand for the product under consideration be the same as if the competing price changed from \$10.00 to \$10.50? Will a change in supply from 25 to 50 units change price by the same number of dollars that a change from 50 to 75 units would change it?

[1] Mills, Frederick C., "The Measurement of Correlation and the Problem of Estimation," this Journal, Vol. XIX, No. 148.

In many cases of this type, the relationships are relative rather than absolute. Changes in business activity might cause *proportionate* changes in price from the existing price. A change from $5.00 to $5.50 in the price of a competing commodity might be no more significant than a change from $10.00 to $11.00 at another time. With regard to supply and price, a large part of the philosophy of the economic calculus is based on proportional rather than absolute changes.

In some cases, such as the ones mentioned, where the change in the dependent variable is a function not of the single independent alone, but of the joint contribution of all the variables, the use of logarithms in the regression equation will state the necessary assumptions in a form which is yet suitable for correlation study. For in a regression equation of the type

$$\log X_1 = b_2(\log X_2) + b_3(\log X_3) + \ldots + b_n(\log X_n) + K. \quad \text{(Eq. 2)}$$

the assumption is that a given proportionate change in X_2, for example, will produce a constant proportionate change in X_1, and further, as is seen by writing equation (2) in terms of the original values of the variables:

$$X_1 = K(X_2)^{b_2}(X_3)^{b_3} \ldots (X_n)^{b_n} \quad \text{(Eq. 3)}$$

the absolute effect of a given value of X_2 upon the value of X_1 will vary with the accompanying values of X_3, X_4, etc.

This form of equation is particularly useful in price analysis because of certain peculiar properties. If in equation (2) we let X_1 represent the price of a given commodity; X_2 the quantity sold in a certain period of time, or the supply reaching market during that period; and let the other X's represent other factors affecting the price, the relation between supply and price, with other factors constant, will be

$$\log_{10} X_1 = b_2 \log_{10} X_2 + K$$

or

$$K' + \frac{1}{b_2} \log_{10} X_1 = \log_{10} X_2. \quad \text{(Eq. 4)}$$

Now according to Marshall,[1] a demand curve representing at every point an elasticity equal to n is

$$\frac{dX_2}{X_2} + n\frac{dX_1}{X_1} = 0, \text{ } i.\ e., \text{ } X_2 X_1^n = C.$$

Expressing this last equation in terms of logarithms

$$\log_{10} X_2 + n \log_{10} X_1 = C'$$

and

$$\log_{10} X_2 = C' - n \log_{10} X_1$$

which is of the same form as (Eq. 4) above.

[1] Marshall, Alfred, *Principles of Economics*, 8th ed., 1920. Mathematical Appendix, Note III, p. 840.

In this equation, n is equal to $-\left(\frac{1}{b_2}\right)$. The regression coefficient b_2 is therefore the reciprocal of the coefficient of elasticity n, which measures the proportion in which the amount demanded increases in consequence of a small fall in the price.

Obviously, as long as the value of b_2 remains the same—as long as the regression of log X_1 on log X_2 is assumed to be linear—the equation is the equation of *constant elasticity of demand*, each small proportionate change in supply bearing a fixed relation to each corresponding proportionate change in price.[1]

The use of this formula in price analysis therefore assumes that the commodity studied is one for which the demand has a constant elasticity. The validity of the assumption may be tested by examining the net regression for curvilinearity by the methods described in a previous paper.[2]

If significant curvilinearity is found, the extent to which the demand does differ from constant elasticity may be determined thereby.

By this method the theoretical demand curve, showing quantities which would be demanded in a given period if various prices were asked, is at least approximated by a curve generalized from different quantities actually sold at different prices at different periods, after eliminating other causes of differences in price. Some error is involved in considering price the dependent and supply the independent, and then using the reciprocal of the regression coefficient to show changes in quantity demanded with changes in price, since only when correlation is perfect will the net regression of price on supply be the exact reciprocal of the net regression of supply on price. However, where the correlation is high the error is slight.

This set-up is also useful in studying prices because it obviates the necessity of "deflating" by dividing by an index of price level before correlating. In place of correlating by the equation

$$\frac{\text{Price}}{\text{Price index}} = A + b_2X_2 + b_3X_3 + \text{ . . . etc.}$$

the equation may be written

$$\log \text{price} = K + b_1 (\log \text{price level}) + b_2 \log X_2 + b_3 \text{ Log } X_3 +, \text{ etc.}$$

The first equation has an implied assumption that there is a 1 to 1 relation between changes in the index of price level and changes in the

[1] This property of the logarithmic relation was pointed out by R. A. Lehfeldt, in "The Elasticity of Demand for Wheat," *The Economic Journal*, Vol. XXIV, June, 1914, pp. 212-217.

[2] Ezekiel, Mordecai, "A Method of Handling Curvilinear Correlation for Any Number of Variables," this JOURNAL, Vol. XIX, No. 148, p. 441.

price of the given article; whereas the second equation leaves the relation to be determined from the value of b_1. This statement of the relations not only makes a much less rigid assumption as to the relation between the price and the "deflator," but makes it possible to test several different "deflators" in turn to see which one gives the closest degree of agreement, while holding constant the relative effects of the other factors included in the correlation.

If curvilinear regressions are obtained for all the variables, the equation in terms of logarithms (in the terminology of a previous paper [1])

$$\log X_1 = K + f_2 (\log X_2) + f_3 (\log X_3) + \ldots + f_n (\log X_n)$$

when stated in terms of absolute values becomes

$$X_1 = K[\phi_2(X_2)][\phi_3(X_3)] \ldots [\phi_n(X_n)]$$

While this makes a rather complex statement of relationships, it still is a very flexible one and carries through the assumption that the variations are all measured in proportionate rather than in absolute terms, and that therefore the absolute effect of a change in any one factor will vary with the absolute values of the other accompanying factors.

Other transformations of the variables in the regression equation, such as the use of reciprocals or complements, may likewise be of value to give expression to other assumptions in certain cases. In any event, all the connotations of the particular equation used should be carefully considered before drawing definite conclusions as to the interpretation to be placed upon the results.

[1] Ezekiel, Mordecai, *op. cit.*, p. 440.

REVISION OF THE WEEKLY INDEX NUMBER[1]

By Irving Fisher

The weekly index number for 1925 differs only slightly from that for 1924. The changes are as follows: Two commodities, butter (state dairy, tubs finest) and hides (country No. 1 steers) have been dropped, because the quotations were not given continuously; and one commodity (branded cows) has been added to the list, the "weight" being 1,000,000,000 pounds.

These changes, applied to the list of 205 commodities given in Table V last year, make 204 as the full [2] number for 1925.

The index numbers for every week, from the first week ending January 5, 1923, to the week ending August 22, 1924, were given in this Journal, September, 1924. Table I continues the series.

Tables II, III and IV give the monthly, quarterly, and yearly averages from the beginning.

As explained in previous articles, while all figures are relative to 100 for 1913, strictly speaking the *base* period for the weekly index in its first year (1923) was not 1913 but the week nearest mid-November, 1922, and the corresponding *base number* was not 100 but 156, chosen because 156 was the United States Bureau of Labor Statistics index number for November, 1922.

Similarly the base period used in the second year (1924) was mid-November, 1923, and the base number, 151.9, that being the index number for that week under the previous (1923) system.

Similarly the third series (1925) is based on mid-December, 1924, and the base number, 155.6, that being the index number for that week by the previous (1924) system.

The reason for using as base for each year a week located several weeks before in the previous year, was to allow time for preparing the revised data for the ensuing year. For this year, the work of revision being so slight, it has been possible to cut down this interval by a month and make the new base not mid-November, as for both previous years, but mid-December.

Thus my index number is really a chain of index numbers, year by year, except that the new base does not begin to be used the very next

[1] The Weekly Index Number of Wholesale Prices was first described in this Journal, September, 1923. The first revision was described September, 1924.

[2] When a quotation is missing in any particular week the number of commodities used for that week is thereby reduced, the omission being, of course, also applied to the base, as explained in this Journal, September, 1923.

week, but a few weeks later. Thus the last two weeks of 1924 were calculated by using mid-November, 1923, as base while the first week of 1925 was calculated by using mid-December, 1924, as base. As a consequence there results, as shown last year, a slight adjustment at the beginning of each year. Thus the index number for the first week of 1925 (ending January 9) is 159.4 whereas it would have been 158.8 if calculated on last year's base, as was the previous week's index.

TABLE I

WEEKLY INDEX NUMBERS

(Continuation of Table I, this JOURNAL, September, 1924, pp. 337–8)

For week ending Friday	Index number of prices in per cent of pre-war, *i. e.*, 1913=100 per cent	Purchasing power of the dollar in pre-war cents, *i. e.*, 1913=100 cents
1924 (*Cont.*)		
August 22	147.1	68.0
August 29	147.4	67.8
September 5	147.9	67.6
September 12	149.7	66.8
September 19	147.3	67.9
September 26	147.8	67.7
October 3	150.5	66.5
October 10	152.5	65.6
October 17	151.9	65.9
October 24	151.3	66.1
October 31	151.5	66.0
November 7	150.9	66.3
November 14	153.8	65.0
November 21	154.8	64.6
November 28	154.4	64.8
December 5	154.6	64.7
December 12	153.7	65.1
December 19	155.6	64.3
December 26	156.7	63.8
January 2	159.1	62.9
1925		
January 9	159.4	62.7
January 16	161.6	61.9
January 23	162.1	61.7
January 30	164.4	60.8
February 6	163.7	61.1
February 13	161.3	62.0
February 20	162.3	61.7
February 27	163.1	61.3
March 6	164.1	60.9
March 13	163.4	61.2
March 20	161.3	62.0
March 27	160.6	62.3
April 3	157.1	63.7
April 10	158.1	63.2
April 17	157.8	63.4
April 24	155.7	64.2
May 1	154.1	64.9
May 8	156.2	64.0
May 15	158.0	63.3
May 22	155.3	64.4
May 29	158.0	63.3
June 5	157.2	63.6
June 12	156.9	63.7
June 19	158.0	63.3
June 26	159.5	62.7
July 3	158.9	62.9
July 10	159.3	62.8
July 17	160.5	62.3
July 24	160.4	62.3
July 31	160.8	62.2

TABLE II

MONTHLY AVERAGES

	Index number 1913=100	Purchasing power 1913=100
1923		
January	157.0	63.7
February	162.1	61.7
March	166.1	60.2
April	167.0	59.9
May	162.5	61.5
June	158.3	63.2
July	153.5	65.2
August	153.9	65.0
September	155.3	64.4
October	155.5	64.3
November	152.0	65.8
December	150.6	66.4
1924		
January	150.5	66.5
February	152.5	65.6
March	148.7	67.2
April	145.9	68.6
May	144.8	69.1
June	143.1	69.9
July	145.2	68.9
August	147.7	67.7
September	148.9	67.2
October	151.8	65.9
November	153.5	65.2
December	155.9	64.1
1925		
January	161.9	61.8
February	162.6	61.5
March	161.3	62.0
April	156.4	63.9
May	156.9	63.8
June	158.1	63.3
July	160.3	62.4

TABLE III

QUARTERLY AVERAGES

	Index number 1913=100	Purchasing power 1913=100
1923		
1st Quarter	161.3	62.0
2nd Quarter	162.6	61.5
3rd Quarter	154.2	64.9
4th Quarter	152.9	65.4
1924		
1st Quarter	150.5	66.4
2nd Quarter	144.7	69.1
3rd Quarter	147.3	67.9
4th Quarter	153.9	65.0
1925		
1st Quarter	161.9	61.8
2nd Quarter	157.2	63.6

TABLE IV

YEARLY AVERAGES

	Index number 1913=100	Purchasing power 1913=100
1923	157.7	63.4
1924	149.3	67.0

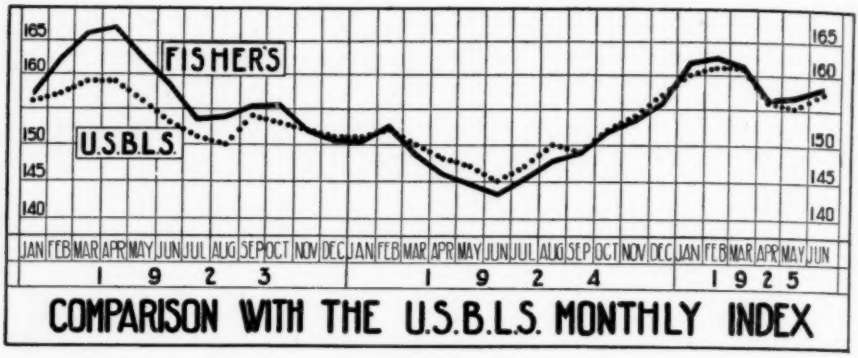


CHART I

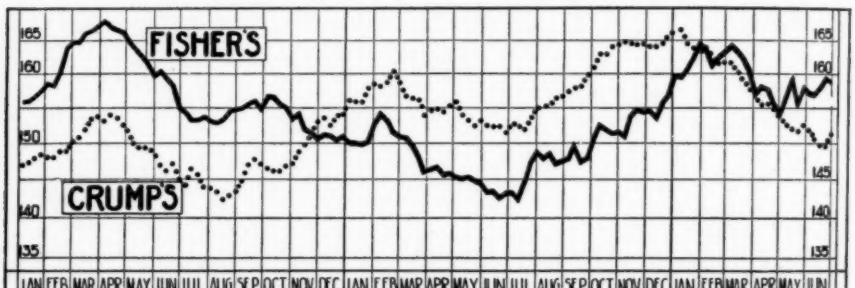


CHART II

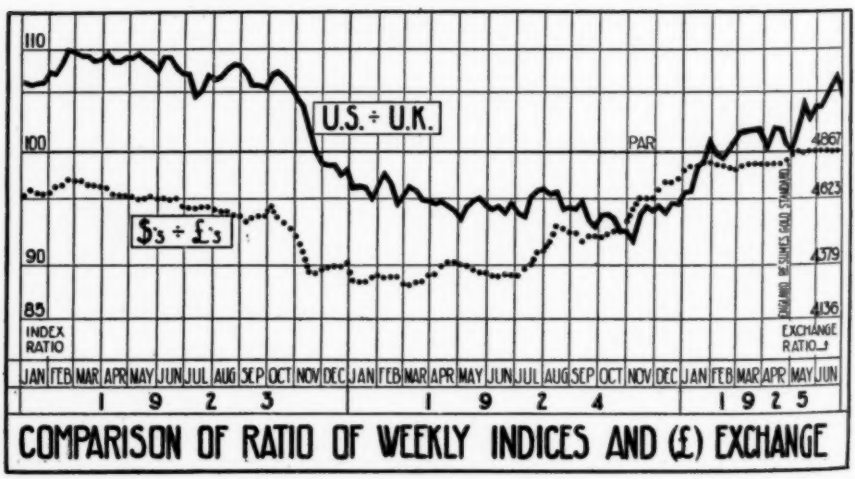


CHART III

It is interesting to observe that the weekly index keeps in close agreement with the monthly index of the United States Bureau of Labor Statistics, as shown in Chart I. The average for December, 1924, is 155.9, or ⅔ per cent less than the 157.0 of the United States Bureau of Labor Statistics. The two indexes keep closer together this year than last and closer last year than the year before. This progressively improved agreement is doubtless due chiefly to the successive incorporation in my index of certain representative commodities not at first available.

Chart II compares my weekly index with Norman Crump's weekly index in the London Financial Times and reveals a surprising similarity in the week-to-week changes despite the difference in monetary standards before April, when England resumed the gold basis. Since resumption, American prices have tended upward and English downward. It would be interesting to discuss why, relative to 1913, our price level is now so much higher than England's.

Chart III compares what Professor Cassel has so well named the "purchasing power parity" [1] of English money (relative to American) with the rate of exchange in New York on London. One of the two curves gives my "U. S." index divided by Crump's "U. K." index, while the other gives the dollars divided by the pounds in cable bills of exchange. The resemblance between the two curves, the former giving the so-called "internal" value of the pound and the latter the "external," is evident, though not striking.

BRITISH PRODUCTION INDEXES

Within recent months measures of the physical volume of production have for the first time been published in Great Britain. Attempts to combine a great variety of series measuring physical output of different commodities into composite index numbers have been rather numerous in this country within the last decade; the results have come to be considered with some degree of confidence and are frequently referred to in economic analyses. Apparently an important influence in making American statisticians pioneers in this field is the greater abundance of statistical information available in this country. Although employing tools by no means inferior, with skill equal to that of the Americans, the British statisticians are considerably restricted by the inadequacy of materials with which to work. These new indexes, however, are undoubtedly an important and valuable contribution as a pioneer undertaking in Great Britain. They provide

[1] As J. M. Keynes has pointed out, the idea, as distinct from the term, antedates Cassel. Ricardo used it. So far as I know the first chart embodying it and analogous to Chart III here, is that in my *Stabilizing the Dollar*, 1920, p. 28.

bases upon which improvements can be made as new data become available and, furthermore, emphasize the need for such additional data.

The new indexes were compiled by Mr. J. W. F. Rowe and were published in Special Memorandum, No. 8 (October, 1924) of the London and Cambridge Economic Service. Revisions and supplementary data have been given in 1925 Monthly Bulletins, Nos. I, IV, and VI. Statistics of net imports of raw materials or exports of finished products provide the chief data for the indexes, and over a period of time are probably fairly accurate measures, because British manufactures import all or nearly all of their most important raw materials and export a large part of many of their finished products. The inadequacy of these data as measures of short-time movements is recognized, and the indexes are presented only on an annual and a quarterly basis. The combined indexes are weighted arithmetic averages of industry relatives. The 1907 census of production, the latest for which information is available, is used for the computation of weights and for estimating the proportion of exports of certain products to the total amount manufactured. The annual index was computed for the years 1907 to 1913 and 1920 to 1923, with the pre-war years as the base period. It includes group indexes of (1) agriculture, (2) mining, (3) iron and steel, engineering, and shipbuilding, (4) other metal trades, (5) textiles, (6) food, drink, and tobacco, (7) chemical and allied trades, (8) paper, printing, and allied trades, (9) leather, (10) rubber, (11) timber, and (12) cement. Most of these groups were in turn subdivided into industry indexes. The quarterly index was computed only for 1913 and the first half of 1914 and for the years 1920 to 1924. The base year is 1913. Of the groups listed above, agriculture, leather, rubber, timber, and cement were not included because of the lack of satisfactory quarterly data, and individual industries from other groups were omitted for the same reason.

The greatest part of the work of compiling these indexes was in securing the basic data, and in many cases the procedure was notably original and ingenious, indicating that a great deal of careful study of individual industries was necessary. For example, in the case of cotton the amount of raw cotton "forwarded to the trade" was reduced to its yarn equivalent to secure a series for the spinning industry. These figures plus imports and minus exports of yarn were then used as indicative of cloth production. A somewhat similar procedure was followed in the case of silk. In several cases a series representing one stage of production was, with due allowance for imports and exports, again used as indicative of a more advanced stage, as, for example, in grain milling and bread and biscuit manufacturing.

Because of the absence of a recent complete census of production and of other comparable statistics a thorough test of the results of these British indexes is practically impossible. Comparisons with employment and railway traffic were only fairly satisfactory. Any lack of agreement was generally explained by differences in make-up. The representativeness of the quarterly index is somewhat dubious because of the omission of certain important industries and because of the inadequacy of raw material import statistics as indicators of productive activity when no allowance is made for warehouse and mill stocks. A further general fault lies in the weights used, as the relative importance of the different industries has probably changed materially since 1907. A surprising

point to Americans, who are accustomed to pronounced and steady growth in production, is that the figures for post-war years are below the pre-war base.

The tables give the actual figures of the British indexes for all periods available. Annual data are also provided for certain of the important groups. The differences between the annual index and the quarterly averages in certain years are due to many variations in the composition of the two indexes—among other things, for example, the influence of agriculture in the annual index. For the

ANNUAL INDEXES OF PRODUCTION

Great Britain—London and Cambridge Economic Service, 1907–13 = 100
United States—Stewart's Index (with post-war estimates), 1911–13 = 100

	Great Britain					United States			
	Combined	Agriculture	Mining	Iron and steel, etc.*	Textiles	All commodities	Farm materials	Minerals	Manufactures
1907	100	100	100	106	102	89	89	86	91
1908	93	100	98	80	90	84	95	79	78
1909	97	102	98	87	99	94	95	87	96
1910	97	101	99	93	91	96	98	94	93
1911	101	98	101	109	101	93	93	93	92
1912	104	100	97	107	108	106	111	101	104
1913	108	98	107	119	108	101	96	106	104
1920	97	89	85	116	76	125	120	131	128
1921	71	76	60	67	58	102	102	103	101
1922	89	93	90	76	80	120	112	106	131
1923	92	91	103	86	68	139	116	148	155
1924	98	95	98	98	73	133	118	135	144

*Iron and steel, engineering, and shipbuilding.

QUARTERLY INDEXES OF PRODUCTION

Great Britain—London and Cambridge Economic Service, 1913 = 100
United States—Federal Reserve Board—Production in Basic Industries, 1919 = 100

Quarters	Great Britain	United States	Quarters	Great Britain	United States
1913	*100*	*96*	*1922*	*81*	*98*
1st quarter	101	99	1st quarter	76	91
2nd "	99	98	2nd "	79	91
3rd "	96	95	3rd "	85	97
4th "	104	94	4th "	83	113
1914	—	*86*	*1923*	*89*	*120*
1st quarter	99	92	1st quarter	89	122
2nd "	93	90	2nd "	89	125
3rd "	—	86	3rd "	85	118
4th "	—	79	4th "	92	115
1920	*100*	*105*	*1924*	*91*	*108*
1st quarter	105	115	1st quarter	90	119
2nd "	101	107	2nd "	91	104
3rd "	102	103	3rd "	90	97
4th "	92	95	4th "	94	111
1921	*68*	*80*	*1925*	—	—
1st quarter	83	83	1st quarter	92	123
2nd "	48	77	2nd "	87	113
3rd "	77	77	3rd "		
4th "	63	84	4th "		

sake of comparisons with the trends in the United States, quarterly averages of the Federal Reserve Board's monthly index of production in basic industries are given, and also Stewart's annual index, and certain of its main groups, with estimates for recent years. All of the indexes for the post-war period show a relatively high rate of output in 1920, depression in 1921, and recovery in later years. This advance was slow and steady in Great Britain, continuing through 1924, whereas in the United States, it was more rapid, reaching a maximum in 1923 and receding in 1924. Comparison of levels in pre-war and post-war years, as indicated above, shows a state of affairs in Great Britain differing widely from that in this country. The British index for every year since 1920 was lower than that for any of the seven pre-war years shown, whereas in the United States only 1921 fell below any of the earlier group of years. Whether the volume of production in Great Britain is actually as much as 10 per cent below the pre-war level, as is indicated by these indexes, must remain a matter of doubt until the figures for the contemplated census of production for 1924 become available.

WOODLIEF THOMAS

Washington, D. C.

A FORMULA FOR FREIGHT RATES

Considered from the cost standpoint, a given freight rate, y, may be divided into two parts, *viz.*, (1) the part properly attributable to carrying the goods over the rails, and (2) the part properly attributable to terminal and other fixed charges. One element varies with the distance, while the other is constant.

This may be expressed algebraically as, $y=m+t$, where y is the rate in cents per pounds, m the haulage variable, and t the constant cost. As will appear presently the haulage costs do not increase at a fixed ratio, but the ratio of increase becomes smaller as mileage increases. This is shown in the following table:[1]

10	miles	cost	4.062	cents	per	ton	per mile.
20	"	"	2.481	"	"	"	"
50	"	"	1.533	"	"	"	"
100	"	"	1.216	"	"	"	"
200	"	"	1.058	"	"	"	"
232	"	"	1.037	"	"	"	"
500	"	"	0.963	"	"	"	"
1000	"	"	0.932	"	"	"	"

The problem, therefore, is to obtain a function of x (mileage) which will increase, but at a regularly diminishing ratio as x increases in value. This may be done by putting[2] m in the form of $\frac{ax}{x+c}$. The algebraic expression of the rate now

[1] United States Bureau of Statistics (Treasury Department), Annual Report 1876, Pt. II, p. 25.

[2] The first idea would be to try a parabola with its axis parallel to the axis of distance. Then the ordinate (*i.e.*, the total rate) will increase indefinitely in decreasing ratio, since the parabola becomes more and more nearly parallel to its axis as we depart from the vertex. The equation of a parabola when the origin is a point on the curve and the x axis is parallel to the geometrical axis of the parabola

becomes $y=\frac{ax}{x+c}+t$ or, clearing of fractions and equating to zero: $xy+yc-ax-xt-ct=0$. Here again y represents the rate in cents per pounds at any given mileage, $\frac{ax}{x+c}$ represents the haulage factor and t the terminal charge. When $x=0$, of course, $mx=0$, and the rate consists simply of the fixed charge. As the value of x becomes greater the haulage factor increases, but in diminishing ratio, approaching a as a limit. This makes a the charge for the maximum distance haul. *In this case* c *is the element which determines the manner in which the haulage charge approaches its limit.*[1] Some insight into the nature of c may be had by considering the rate of change in the haulage charge. As indicated, this is not constant, but at any distance x is given by $\frac{d}{dx}\left(\frac{ax}{x+c}\right)=\frac{ac}{(x+c)^2}$; therefore, the rate of increase per mile of the haulage charge is inversely proportional to $(x+c)^2$ and therefore diminishes rapidly as distance increases.[2]

To illustrate the manner in which this formula may be used in deriving a complete distance scale consider the following hypothetical case. Assume terminal costs, plus fixed charges aggregating 21 cents per hundred pounds; assume also that at 1,000 miles the cost of haulage is $1.20, and that at 475 miles it is $.80. Substituting these values in the formula $xy+cy-ax=0$ results in the following simultaneous equations:

$$1000(1.2)+(1.2)c-1000a=0$$
$$475(.8)+(.8)c-475a=0$$

which, when solved, yield the following constants:

$$a=2.19$$
$$c=825.$$

From these constants the rates for any number of miles can be determined as follows:

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is $y^2+2ay=4px$ where p is the distance from the vertex to focus. Solving for y and adding t, the terminal charge, we have the total rate, $y=-a+\sqrt{4px+a^2}+t$. The hyperbola is simpler to work with and for distances up to and including the maximum range to which rate schedule applies is equally satisfactory. If $y=\frac{ax}{x+c}$ and the total rate be $y=\frac{ax}{x+c}+t$, y is a function of x which not only increases more and more slowly as distance increases but approaches a definite limit as the distance becomes indefinite. While this may seem an objection since the rates calculated for very large distances may differ by small amounts, yet for distances to which the schedule is intended to be applied the difference is slight and the fit in favor of the hyperbola.

[1] Let $x=c$, then $\frac{ax}{x+c}=\frac{a}{2}$, now c becomes the mileage distance at which the haulage cost is one-half the cost for the longest possible haul.

[2] This agrees with Professor Ripley's statement (*Rates and Regulation*, p. 103), "A common rule is that the rate rises as the square root of the distance rather than in proportion to it."

(1) x	(2) $2.19\times(1)$ ax	(3) $825+(1)$ $x+c$	(4) (2)/(3) $\frac{ax}{x+c}$	(5) (4)+.21 $\frac{ax}{x+c}+b$
20	43.80	845	$.0518	$.262
40	87.60	865	.1013	.311
60	131.40	885	.1485	.358
80	175.20	905	.1936	.403
100	219.00	925	.2367	.446
120	262.80	945	.2781	.488
140	306.60	965	.3177	.527
160	350.40	985	.3557	.566
180	394.20	1005	.3939	.603
200	438.00	1025	.4273	.637
220	481.80	1045	.4610	.671
240	525.60	1065	.4935	.703
260	569.40	1085	.5248	.735
280	613.20	1105	.5549	.765
300	657.00	1125	.5840	.794
320	700.80	1145	.6120	.822
340	744.60	1165	.6392	.849
360	788.40	1185	.6653	.875
380	832.20	1205	.6900	.900
400	876.00	1225	.7150	.925
420	919.80	1245	.7390	.949
440	963.60	1265	.7620	.972
460	1007.40	1285	.7840	.994
475	1040.25	1300	.8000	1.010
480	1051.20	1305	.8060	1.016
500	1095.00	1325	.8260	1.036
600	1138.00	1425	.9220	1.132
700	1314.00	1525	1.0005	1.215
800	1752.00	1625	1.0780	1.288
900	1971.00	1725	1.1420	1.352
1000	2190.00	1825	1.2000	1.410

This formula is also useful in analyzing current rate schedules into their respective elements. As an example it is desired to know the terminal charges and haulage costs involved in the distance scale proposed by the carriers in 1922 for Trunk Line territory. At 20 miles the rate is 36 cents, increasing to 44 cents at 40 miles, then at a flat increase of 4 cents per 20-mile block up to 160 miles, a 3-cent increase to 180 miles and from 180 to 500 miles a flat increase of 2 cents per 20-mile block.

The formula is $y=\frac{ax}{x+c}+b$, which may be written, $xy=x(a+b)+bc-yc$, and normal equations, by the method of moments, for fitting the line, are as follows:

$$\Sigma xy=(a+b)\Sigma x+bcn-c\Sigma y$$
$$\Sigma x^2y=(a+b)\Sigma x^2+bc\Sigma x-c\Sigma xy$$
$$\Sigma x^3y=(a+b)\Sigma x^3+bc\Sigma x^2-c\Sigma x^2y$$

The summations, by computation, are:

$\Sigma x=6500$, $\Sigma y=19.08$, $\Sigma xy=5631.4$, $\Sigma x^2=2{,}210{,}000$, $\Sigma x^2y=2{,}012{,}780$, $\Sigma x^3=845{,}000{,}000$, $\Sigma x^3y=792{,}073{,}840$.

Substituting these values in the normal equations,

$$5613.4=6500(a+b)+25bc-19.08c \quad (1)$$
$$2012780.=2210000(a+b)+6500bc-5613.4c \quad (2)$$
$$792073840.=845000000(a+b)+2210000bc-2012780c \quad (3)$$

and solving the simultaneous equations the following constants are found,

$$a = 1.48$$
$$b = .352$$
$$c = 614.26$$

These constants mean, (1) that the maximum haulage rate that could be charged under this form of distance scale would be $1.48 and that would be at an infinite distance; and (2) that the terminal and other fixed charges amount to $.352.

Reversing the process and calculating a distance scale using the above constants in the formula practically reproduces the original rates, thus proving the consistency of the method.

The utility and importance of this formula are obvious. At the present time two major rate investigations are being conducted by the Commission and further investigations of greater comprehensiveness are planned. By means of this formula direct comparison of the component parts of various rate structures is possible. Again, terminal costs are readily ascertainable, and haulage costs for varying distances can be measured within a reasonable approximation.[1] Upon these two factors schedules of rates can be developed which bear a precise relationship to the facts of transportation.

JOHN G. SCHAFFER

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PROFESSOR COMMON'S REVIEW OF "THE TAX PROBLEM IN WISCONSIN"

To the Editor of the Journal of the American Statistical Association:

Sir:

In the March, 1925, issue of this JOURNAL (pp. 138–139), Professor J. R. Commons reviewed a recent investigation of the National Industrial Conference Board, entitled *The Tax Problem in Wisconsin.*

Professor Commons makes sweeping assertions of one-sidedness, partiality to a particular interest and unreliability which ought, in all fairness, to receive definite proof to support them. What is the nature of the data that Professor Commons offers in support of his sweeping statements?

He states first that the Board's report "makes no use of a readily available source of information as to which classes of the community are carrying the heavier tax burdens, namely, the income tax returns, which in this state are not subject to a secrecy clause." He then points out that "a sampling investigation, not at all complete," by Professor Ely showed that the state tax burden on manufacturers is only about two-thirds as great as it is on farmers. This broad

[1] Commenting on transportation costs as early as 1910 the late Commissioner Franklin K. Lane said, "It was regarded as practicable by statisticians to leave but a narrow 'twilight zone,' between the actual cost of moving a ton of freight and the statistician's estimate, and this estimate it was thought would not vary 5 per cent from the total cost." (*Advances in Rates*, Western Case 20 I. C. C. 307).

statement embodies a new lesson in logic for statisticians! An inquiry which is described by Professor Commons himself as "a sampling investigation, not at all complete" and about which it is said "if these figures hold good for all of the state" is presented as "on the face of it" overthrowing "the main contention of the National Industrial Conference Board." It is a huge "*if*" that Professor Commons so lightly hurdles.

The investigator of the Conference Board did look very carefully into the data suggested by Professor Commons. It was found that in 1922, which is the latest year for which income tax returns have been compiled, only 12,660 farmers were assessed with an income tax, although it is likely that the number who paid an income tax that year was much smaller because of the deduction of the personal property tax. According to the Census of Occupations in 1920, there were 290,268 persons eighteen years of age and over engaged in agriculture in the state of Wisconsin, and it is probable that by 1922 this number had increased because of the profitableness of dairy farming in post-war years. The Census of Agriculture placed the number of farms in Wisconsin at 189,295 in 1920, but the number of farmers with tax-paying ability may be considered in excess of this figure. It is clear, therefore, that even if every income tax return were examined in the case of farmers, these would cover only a fraction of the total number of farmers, as compared with a virtually full coverage for industry, trade, public utilities, etc. Furthermore, it is a well-known fact that but few farmers keep books, that there is no means of readily verifying tax returns, and that farmers fail to include in their income a large part of the food produced on the farm but consumed at home. How an examination of such defective data could overthrow the findings in the Board's study is beyond comprehension.

Furthermore, even if the income tax data were otherwise acceptable, one is still confronted with the problem of tax incidence. The farmer who pays the general property tax at the present time—and this constitutes the bulk of his tax burden—does not necessarily bear it. Those who have bought the land from its original owners have capitalized the preëxisting taxes and anticipated somewhat future increases. This process of capitalization has continued from the first moment that property changed hands from its original owner. Is it correct to relate the total taxes the farmer pays now to his present income, were it ascertainable, when as a matter of fact a large part of his present taxes has been capitalized into a lower purchase price paid for the property?

Professor Commons refers to the situation in Ohio. Apparently he is relying largely upon general impression, as others have done in depicting the situation there. In 1912, the per capita net debt of Ohio was three times as high as in Wisconsin ($48.27 as compared with $16.56). No question was then raised as to the ability of Ohio to stand this burden. In 1922, the Census Bureau figures indicate that the relationship was slightly less than three-fold—specifically an increase of 132.9 per cent in the case of Ohio, as compared with 134.4 per cent for Wisconsin. The increase for Illinois was 131.4 per cent, and for Indiana, 109.8 per cent. Of the surrounding industrial commonwealths, only Michigan showed a higher rate of increase than Wisconsin. Debt incurrence is not to be deplored if the proceeds are spent wisely and productively. In fact, when the object of

the expenditure is of the nature of a permanent improvement and when the general tax burden continues to remain high, would it not appear advisable to modify the pay-as-you-go policy?

The reviewer takes issue with respect to the efficacy of methods of evading the income tax by intercorporate devices. A more careful reading of the statements in the Board's text would undoubtedly have obviated the comments made. If a domestic corporation owning a factory in Wisconsin sells its plant to a foreign corporation and the main offices and sales force are transferred outside Wisconsin, and if the product is then sold at nominal cost to a sales corporation which acts as an intermediary, evasion of the income tax is effected. "An examination of the statutes or a five-minute inquiry at the office of the Tax Commission," as suggested by Professor Commons, cannot alter the fact. He may rest assured that the Board's chief tax investigator spent many hours and days with various members of the Tax Commission and consulted with income tax lawyers and tax experts before the statement was formulated as set forth in the Board's report. While a "five-minute inquiry at the office of the Tax Commission" might have proved a reply satisfactory to Professor Commons, a few hours' talk with accountants of the Tax Commission, supplemented by careful study of cases and conferences with those who are familiar with the practice of income taxation, has indicated otherwise. Legislatures may be "clever or stupid," as the case may be, but there are interstate legal obstacles which even a clever legislature cannot overcome.

The Board hopes that serious-minded students interested in these tax data will examine this important investigation of the Board and determine for themselves the degree of accuracy, scientific care and honesty employed in it. The Board is of the opinion that the sweeping statements of Professor Commons are not justified by the supporting data which he presents in his review and that the review is neither fair nor helpful to anyone interested in the zealous pursuit of fact.

Very truly yours,

L. R. GOTTLIEB

AN ATTEMPT AT A LOGICAL APPROACH TO CERTAIN FUNDAMENTALS IN PHYSICAL MATHEMATICS

The present suggestions are more strictly mathematical than statistical, but the processes of reasoning involved are primarily those of the statistician, which fact may establish at least a tolerance for the publication of the argument in the pages of the JOURNAL.

Modern thinking in the field of cosmic mathematics and cosmic origins is quite ready to find that the cosmos, as a whole, is a matter, primarily, of mathematical relations and mathematical formulae.

From this point of view it is but a short step to that of assuming a form of cosmic evolution which has its origin in certain underlying necessities which are both logical and mathematical in their character.

The analysis of physical phenomena which develops from this latter point of view, with its applications to problems such as those of gravitation, cohesion, the assumed Lorenz contraction, etc., is inappropriate to the present brief discussion. There is, however, an element in this analysis which can be briefly stated and may be of interest to those who deal regularly with frequency distributions and the theory of probabilities. This particular element concerns itself with the origin of fundamental measures of velocity and extension—which measures necessarily carry with them a derived measure of time.

For the purpose of this special analysis, it is first necessary to assume that any coherent physical plan must involve the existence of certain prime elements which possess the characteristics of extension in space and of motion.

The second assumption is that, since these elements are the *prime* elements and their motion is the *prime* motion, the prime physical measures of velocity and extension must be based upon their dimensions and their rates of motion.

Furthermore, since there are no preëxisting measures of velocity or extension to which these prime elements can in any way be subject or related, it is possible to make the third assumption, which is that, both as to dimensions and as to rates of motion, these elements are arranged in frequency distributions of indeterminate range, which are entirely free from any tendency toward modal concentrations and are subject only to accidental variations from absolute uniformity in the number of the prime elements which lie within any equal intervals of range throughout the whole spread of the distributions.

With these three assumptions once made, the succeeding steps are relatively simple. The prime measure of velocity must necessarily arise from the prime velocity distribution, and, similarly, the prime measure of extension must arise from the prime dimensional distribution. In each case the natural unit of measure is the mean—and the special characteristic of the uniform type of distribution which has been assumed to be logically necessary is that the mean is always one-half the maximum, *whatever that maximum may be.* In other words, the maximum may be infinity, but, if this infinity is measured in terms of a unit which itself is one-half of infinity, then this pseudo-infinity, in terms of that unit of measure, becomes mathematically equal to *two.*

At this point it becomes desirable to make a mathematical distinction between indeterminates and true infinities. The maximum velocity in the indeterminate range which has been discussed may still be able to move in a straight line for a true infinity of time without passing beyond the limits of space—and the simple assumption that there is a truly infinite number of the prime elements carries with it the correlative assumption that, however large in an abstract sense the maximum element may be, there still must exist a true infinity of space beyond.

For the purposes of these brief notes there is little to add to what precedes. The logical steps which have been indicated are in such form that they lend themselves readily to correction or modification. Their acceptance, if they stand the final tests, will not limit the field of the physicist in approaching, in his own more careful and certain way, the solution of physical problems. But much might be gained if it could once be established as a physical law, or even as a promising working hypothesis, that the special mathematical conditions which

have been indicated would exist in those cases where prime measures had to be derived from the magnitudes to be measured.

The hypothesis as it stands would, in fact, seem to offer a very direct explanation of the limitation of the velocities of material particles and of the special applicability of the pseudo-spherical geometrical frame-work to the development of certain formulae of mathematical physics.

M. C. RORTY

NOTES FROM THE CENSUS BUREAU

A CENSUS OF DETROIT

The city of Detroit has just completed a population census which was taken under the supervision of a representative of the Bureau of the Census, Mr. G. B. Weitzel. A state law in Michigan requires an annual census of school children, on the basis of which the state school fund is apportioned among the municipalities. The city officials of Detroit, having decided to expand the school census this year so as to cover the entire population, sought the coöperation and advice of the Census Bureau, which agreed to detail a representative to supervise the work.

The schedule used was a family schedule, a separate slip being filled out for each family carrying the names of all members of the family. The data called for included sex, age, country of birth, and citizenship, with some additional facts for school children as to school attendance, employment, and physical defects.

The city was divided into 571 enumeration districts. The majority of the enumerators were school teachers; and the district supervisors, 53 in number, each having charge of from 8 to 15 enumerators, were school attendance officers. The enumeration was begun on May 12 and was substantially completed by June 10.

The enumerators were paid at the rate of 4 cents a name for each child and 2 cents for each adult. The estimated cost of the census including tabulation is $55,000 of which $35,000 represents the cost of the regular school census defrayed out of the school funds. The additional $20,000 required for taking a complete census was provided from the funds of the Water Board, the Police Department, and the Board of Health.

This is by no means the first instance in which a municipal census has been taken by the municipality under the supervision of a representative of the Bureau of the Census. But it is the first time that it has been done for as big a city as Detroit, and it is the first time that such a census has been combined with the local school census. There are many cities which regularly take a school census and it is to be hoped that the Detroit census may prove to be a precedent for taking similar censuses in other cities, especially those where the conditions of growth have been so exceptional or peculiar that the rule by which the Census Bureau estimates the population is not applicable, and in consequence no estimates are made. Detroit came into that class. The Bureau has made no estimate of its population since 1920, and therefore has published no death,

birth, or other rates for that city. Other large cities for which estimates have been discontinued are as follows:

Akron, Ohio	Los Angeles, California
Atlanta, Georgia	Norfolk, Virginia
Bridgeport, Connecticut	Oklahoma City, Oklahoma
Elizabeth, New Jersey	Seattle, Washington
Erie, Pennsylvania	Waterbury, Connecticut

It is generally recognized that there ought to be a complete but simple population census of the United States in the fifth year following the decennial census. During the first half of the decade the estimates are as a rule sufficiently accurate for the purpose for which they are used. But they become increasingly uncertain and unreliable in each later year. The quinquennial census may come some day. In the meantime the next best thing is the taking of local censuses by states and municipalities and this is something which the Bureau of the Census wishes to encourage in every practicable way.

The following states regularly take a census in the fifth year following the decennial census, and have taken a census within the last six months:

Florida	New York
Iowa	Rhode Island
Kansas	South Dakota
Massachusetts	Wyoming

It is desirable that the state censuses should follow the methods and principles of the Federal census, so that the results may be comparable; and with that aim in view the Director of the Census has endeavored to get in touch with the state officials and coöperate with them. But while the enumeration in some states may not meet the standards of the Federal census, nevertheless the policy of the Bureau will be to accept the results of the state census so far as possible and use them in place of estimates.

CHANGES IN PERSONNEL

On account of ill health, Mr. William C. Hunt has resigned the position of Chief Statistician for Population in the Bureau of the Census. Mr. Hunt was connected with the Massachusetts state censuses of 1875, and of 1885, and with the Massachusetts Bureau of Labor from 1883 to 1889. His connection with the United States Census dates from the year 1889. He was made Chief Statistician for Population in 1899. He had charge of the population censuses of 1890, 1900, 1910, and 1920, of the marriage and divorce inquiry of 1906, and of the census of religious bodies taken in 1906 and again in 1916. He wrote many of the census reports on population and other subjects. In collaboration with Carroll D. Wright he prepared the volume on History and Growth of the United States Census, published by the Department of Labor in 1900.

The vacancy in the position of Chief Statistician for Population in the Bureau of the Census created by the resignation of Mr. William C. Hunt has been filled by the appointment of Mr. Leon E. Truesdell.

Mr. Truesdell graduated from Brown University in 1907. From 1911 to 1914 he was on the editorial force of the Bureau of the Census in the division of Revision and Results. From 1914 to 1919 he was employed as agricultural statistician in the Bureau of Markets and Rural Organization in the Department of Agriculture, working on rural credit problems and rural social surveys. In 1919 he returned to the Bureau of the Census as an expert special agent; and in that capacity he has had experience in planning and supervising some of the important census tabulations. Mr. Truesdell wrote a bulletin on "Amortization Methods for Farm Mortgage Loans," published by the Department of Agriculture. He collaborated with Dr. E. A. Goldenweiser in preparing the census monograph on "Farm Tenancy in the United States," and is the author of the monograph on "Farm Population," which is yet to be published in the same series.

J. A. H.

ANSWERS TO QUESTIONS INVITED

By the Advisory Committee of the Bureau of the Census

The Advisory Committee of the Bureau of the Census wishes to ascertain what is the best practice in publishing death-rates or birth-rates based on a small population or a small number of deaths or of births. Published reports from various countries show no uniform practice regarding such rates.

One report shows rates based on a single death, even where the population is less than 1,000.

One omits rates "because the population is so small (21,364 in 1921)."

One omits rates where the population is less than 100.

One shows rates in italics where deaths number 4 or less.

One shows ratios of births to deaths only where the deaths number at least 100.

The Advisory Committee therefore invites replies to the following questions:

1. What is the lower limit of population enumerated or estimated, if any, for which it is desirable to compute and publish a death-rate or a birth-rate?

2. What is the lower limit of the number of recorded deaths or births, if any, for which it is desirable to compute and publish a death-rate or a birth-rate?

3. Is it good practice to distinguish between rates based on an insufficient population or an insufficient number of births and deaths and those for which both bases are sufficient by some such device as printing the former in italics?

Answers to the preceding questions or any of them should be sent to the Director of the Census at Washington for transmission to the Committee.

(Signed) JOINT ADVISORY COMMITTEE OF THE AMERICAN ECONOMIC ASSOCIATION AND THE AMERICAN STATISTICAL ASSOCIATION TO THE DIRECTOR OF THE CENSUS

By the Committee on Educational and Professional Standards for Statisticians

During the spring months Dr. W. I. King, Secretary of the American Statistical Association, sent out to American colleges and universities the questionnaire form which was prepared by the Association's Committee on Educational and

Professional Standards for Statisticians. The purpose of the inquiry was to determine the nature and extent of statistical instruction in colleges and universities. At the present time, a considerable number of the questionnaires have been returned. A summary of the returns will be made and presented at the annual meeting of the Association. Teachers of statistics who have not yet had an opportunity to report are requested to communicate with the Office of the Secretary and to secure a copy of the questionnaire.

MISCELLANEOUS NOTES

Meeting on the Measurement of Seasonal Variations.—A dinner meeting of the American Statistical Association, at which 103 persons were present, was held on Friday evening, May 22, 1925, at the Aldine Club, 200 Fifth Avenue, New York City. The subject under discussion was "The Measurement of Seasonal Variations." President Robert E. Chaddock was compelled to be absent, as he was at Washington serving as a member of the Advisory Committee on the Census. He telegraphed his regrets. Dr. Harry Jerome of the National Bureau of Economic Research acted in his place as presiding officer.

The first speaker was Prof. William L. Crum of Harvard University. He stated that, of the many devices for measuring seasonal variations, there were only four which have had any considerable support; the monthly mean method, the Persons median-link-relative method, the Macaulay median-ratio-to-moving-average method, and Miss Falkner's ratio-to-ordinate method. All of these were summarized in Professor Falkner's article in the June, 1924, JOURNAL. Mr. Crum called attention to the fact that, in the September, 1924, JOURNAL, Willford I. King had opened the question of varying seasonality, and expressed the view that this was an important addition to current thought on the matter. However, he believed that the method used by Mr. King in measuring seasonal variations was quite erroneous and misleading. Mr. Crum showed that seasonal fluctuations might vary either gradually or suddenly, and that there might be cyclical changes in seasonal variation. However, he called attention to the fact that certain methods of measuring seasonal changes might indicate cycles in the seasonal when none really existed. Such errors would arrive from giving an excess correction during the period of depression, or vice versa.

The first major objection that he expressed to the method used by Mr. King was that it did not prove itself when reversed. He stated that, if the nine-year moving median were applied to the percentage deviation from the final cycle curve as recorded by Mr. King in his article in the September JOURNAL, the seasonal variations arrived at would not be the same as those finally arrived at in Mr. King's study, for the reason that the median would cut off the turns at a number of points. Since the smoothed curves from which the final readings of the seasonals were taken were derived from the moving medians, he felt that it was necessarily fallacious to assume that they could be more accurate than the data upon which they were based. Mr. Crum said:

> To begin with, consider the individual median and its precision. Examination of each of the nine-year distributions upon which Mr. King's unadjusted medians (I am using the term unadjusted for the median derived directly from the data as distinguished from the adjusted median which he gets from his curve in Chart 2) shows in almost no case that close concentration of variates which is essential to the proper utilization of the median as an average. There are certain very

excellent objections to a discussion of the probable error in a problem such as this, but if we keep in mind the necessary reservations, such discussion will shed important light of a descriptive character upon the medians and their precision. To estimate the probable error for any particular nine-year moving average, the upper and lower quartiles are found by taking respectively the means of the seventh and eighth and of the second and third arrayed variates, and computing one-half the inter-quartile range. This is a fair approximation to the probable error. The next step is to estimate the probable error of the difference between the two medians, utilizing the individual probable errors incident to the determination of each median. Finally, this probable error of the difference is compared with the difference itself. The point I am driving at is to ascertain by consideration of the probable errors whether the difference between the unadjusted median, applying to a particular month, say January, for one year and the corresponding number for some other year as found by Dr. King's method, has any significance or may reasonably be assumed as due to chance. The most favorable comparison to Dr. King's position is likely to be a comparison involving the maximum change of the median for any particular month. In these cases of maximum change it appears on the whole that the amount of change does not considerably exceed the estimated probable error. Keeping in mind essential reservations about the applicability of probable error doctrines to series ordered in time, and also the admittedly approximate method used in estimating these probable errors, I should say that even for the most favorable cases the difference shown by Dr. King's unadjusted medians is not sufficiently greater than the corresponding probable error to give us any considerable confidence that such difference arises from any other cause than chance. When we pass to the less favorable cases where the change in the unadjusted median is notably less than the maximum (and these cases are after all those in which we are primarily interested, for it is only these changes which can well arise in the so-called short-time fluctuations in seasonality) the situation is decidedly less convincing. There is much less reason for confidence that the differences discovered have any significance. There is decidedly more reason for believing that such differences as are found arise solely or at least predominantly from chance. On this ground, then, essentially that the dispersions of the nine variates used in getting each median is so great that the precision of such median and of the comparison based upon two such medians is slight, I would urge that Dr. King's nine-year moving device is unsound and yields unadjusted medians which show short-time variations which are largely, if not predominantly, illusory.

Before passing to the consideration of a possible objection to the point just made, I should like to present further evidence of the lack of precision of the individual medians. Examination of a few (or for that matter a great number) of the individual nine-year distributions upon which the unadjusted medians are based will disclose the fact that in nearly all cases the median is isolated from its neighbors. The nearest variate to the median in many cases is several whole per cent distant. This emphasizes the fact that here is a problem in which the median is based upon so small a number of variates which are so widely scattered that it has that erratic character which has frequently been noted. Only when there is sufficient concentration of the individual variates about the median is the median fixed narrowly and incapable of erratic fluctuations. In many of Dr. King's distributions the number which happens to be the unadjusted median would still be the unadjusted median if it were shifted 5, 10, 20, in some cases 30 tenths of a per cent higher or lower. Surely such a measure of the average of a series is not sufficiently precise to justify its expression in tenths of a per cent.

It will be said of all this, however, that we are not concerned, in the practical application of Dr. King's method, with the unadjusted medians or indeed with any unadjusted average for the individual nine-year distributions. It will be said rather that we are concerned with adjusted medians based upon a simultaneous consideration of several adjacent and successive nine-year moving distributions, that the adjusted medians so found, for example by means of his curves in Chart 2, are a much more highly reliable group of measures than the unadjusted medians about which we have been talking. In answer to this it may be noted in the first place that it is a dangerous doctrine to assume that the more complex and the more intricate statistical operations yield the more reliable results. It is a treacherous thing to get so far away from our data that we lose

sight of the inherent irregularities and disturbing characteristics of the original items themselves. No sound reason appears for believing that consideration of a group of distributions all together, each of which is itself highly dispersed, will necessarily furnish a series of measures which can be highly precise. You cannot get greater accuracy out of data than is there to begin with. Statistical method does not supply accuracy to data. No statistical method can improve our material. Faulty, highly dispersed, grossly irregular data cannot yield closely typical and numerically precise summary measures.

Moreover, in the present case, the problem, if it is looked at from this point of view, is fundamentally a problem of regression. In the case of his January ratios, Dr. King is getting a line of regression when he draws his January curve of Chart 2. Now we have certain criteria of the appropriateness of curves or lines of regression. One of the essential things for a highly precise curve of regression is that the clustering of the individual variates, as represented in the scatter diagram, along such curve shall be close. What is said above indicates that such clustering cannot be close in this problem. The dispersion of the individual variates for a particular nine-year group from the median of that group is large. Then the dispersion of the variates of several such groups from a curve roughly through the medians of such groups will also be large. The form and the position of such curve cannot confidently be stated with any considerable degree of arithmetical accuracy. Examination of a correlation diagram for any one month of Dr. King's data will show that the scattering of the plotted points from his curve of adjusted medians is so wide as to render doubtful all of the minor swings of such curve.

I would raise another point here, although it is not essential to this phase of the argument, and that is this: that this is a very exceptional correlation table, a table in each vertical array of which all but one variate is identical with variates in the adjacent vertical array on either side. This is an essential consequence of the moving median idea. Perhaps this confusing condition could be removed from the problem by considering the correlation table, not of the groups of ratios involved in the nine-year moving median process, but merely the original ratios, one ratio for each vertical array. More will be said on this point shortly.

My conclusion from the consideration of the regression aspect of the method is that the utilization of any form of regression curve, except the very simplest kind, is not justified. A straight line or some very simple curve, such as a parabola, are about the only possibilities which could be supported in the light of the doctrine of correlation. The utilization of such a considerably distorted curve as Dr. King has for March or November or December in his Chart 2 can scarcely be justified on these grounds. I would suggest that in almost all cases the only regression curve which is feasible in this problem is a straight line. It is true that there will be some departures from such straight line which seem to be systematic. There will be portions of the straight line for which the majority of the variates tend systematically to lie on one side rather than on the other. This tendency, when considered in the light of the great dispersion within the vertical arrays, is not sufficiently pronounced to warrant the utilization of complicated curves of regression.

The same conclusions are to be drawn from an examination of the regression problem as applied to the individual ratios rather than to the nine-year moving groups. Here also the only sound results which can be derived from available data are with one or two exceptions straight lines of regression. There is no satisfactory evidence that the regression relationship in any month partakes of the wavy and rapidly changing form shown by some of Dr. King's curves in his Chart 2.

All of these considerations seem to point to the conclusion that the material available justifies our seeking progressive variation in seasonality in this problem, but not short-time variation in seasonality. If we grant that there may be progressive variation in seasonality in the daily rate of pig-iron production, I contend that the most satisfactory way of testing its existence and measuring its amount is by the utilization of the link-relative device as elaborated in my discussion in the March, 1925, JOURNAL. I suggested that, for cases in which the movement was sufficiently irregular to be at all in doubt, the most sensitive way of making the test was by utilization of Professor Person's link relatives. I have computed, therefore, for this series the link relatives and have constructed

for each month a chart of ordered data analogous to Chart 1 of my JOURNAL article. In no one of these charts is there any considerable evidence that the fluctuations in the link relatives are of the short-time character. In nearly all of the charts there is evidence that the only change is of a progressive character which can be described adequately by a straight line; and in only one or two exceptions does the progressive change have the form of a curve, and then a simple curve.

Mr. Crum's second principal objection to the King method of measuring seasonals was that it required the use of discretion on the part of the investigator. He felt that Mr. King was distinctly in error in assuming that a clerk competent to handle this procedure after a few hours' instruction could readily be found. In his opinion, individual judgment should enter into a statistical problem only in selecting the method of approach, and not in the application of the method. It was his view that the seasonals for pig-iron production found by Mr. King for the various months probably were mainly dependent upon the way that the original cycle curve happened to be drawn, rather than upon any seasonal fluctuations in the data themselves.

The second speaker was Prof. William A. Berridge of Brown University. Mr. Berridge made a most careful comparison of the essential features of several of the most commonly used methods for measuring seasonals. He showed that, in many of the earlier methods used in measuring seasonals, no effort had been made to eliminate the trend, but that most of the present students of the subject believe that this step is essential. The Falkner method gets rid of the trend at the beginning by taking as a basis the ratio of the item to the trend. The Persons method does not eliminate the trend at the outset, but finally removes it from the link-relative ratios. The methods proposed by Mr. Macaulay and Mr. King each attempt to eliminate both trend and cycle at the outset. He pointed out that the inclusion of the cycle in the original ratios, as determined by the Falkner and Persons methods, constitutes a weakness in each. He also brought out the fact that the moving average was not ideal as a measure of the cycle, because it did not conform accurately to the sharper curves of the cycle. Mr. Berridge further expressed the view that the present tendency was to devote attention to the changing nature of seasonal movement, rather than to assume that it remained constant over a long period of years.

Owing to the illness of Mr. Harold M. Flinn of the American Telephone & Telegraph Co., his paper was read by Mr. Joseph P. Shaw, Jr., also statistician for the same organization. Mr. Flinn emphasized the need of using as far as possible the inductive rather than the deductive method, and of employing in the practical applications of statistics the simplest devices that will work. He stated that his investigations have demonstrated that seasonal fluctuations tend to be proportional to the sizes of the respective items rather than absolute in amount, and that experience shows that in practice the seasonal ratio does not remain constant, but is inclined to vary as time passes. Sometimes the changes are gradual—sometimes sudden and abrupt. Abrupt changes are frequently caused by the enactment of new laws or by new discoveries in technique. Gradual changes are, however, more common. In many cases, there is a tendency for seasonal movements to become less in amplitude as the years pass. He stated that for several years he and his associates have been using a procedure which is a modification or extension of the link-relative method originated by Prof. Warren M. Persons. This revised method begins by determining link relatives in the same way as in the Persons method. From that point the method diverges. He explained the procedure as follows:

The first step consists in plotting the link relatives for each month and examining the trends of the twelve resulting curves. It has been our experience in the

analysis of a large number of series that the trends of the link relatives are not well adapted to the fitting of simple analytic functions.

In the first place, it has been found from scrutiny of many series that the trends are extremely diversified. For one month it might appear that the straight line would afford a good fit. For another month, a second degree parabola or an exponential curve would appear to be appropriate. And for still other months, in order to secure a fit that could be called at all reasonable, it would seem to be necessary to make use of curves of a much more complex nature.

In the second place, the use of analytic functions fitted by the method of least squares or the method of moments would tend to defeat one of the principal advantages of the link-relative method mentioned above, namely the use of the median as the typical item. For a curve fitted by either of the methods mentioned partakes largely of the nature of an arithmetic mean. Its position and slope are affected to a very appreciable extent by the irregular or extreme items. And since in most series of economic data the presence of such items is of common occurrence, fitted curves would reflect the effect of unusual influences as well as of seasonal factors. This result is certainly to be avoided.

And in the third place, the fitting of analytic functions would tend to be unnecessarily laborious. The use of such functions implies an empirical law which might reasonably be expected to continue to fit future data as they are recorded. Here again, it has been a matter of experience that additional data for three or four years may change the nature of the curve to a great degree. The link relatives for a given month may, over a period of ten to twenty years, exhibit two or three pronounced maxima and minima; and a declining movement for the past six or eight years is by no means certain to persist for the next three or four years.

For these reasons, the fitting of analytic functions is regarded as unsatisfactory. It becomes necessary, therefore, to find a means of describing the data during the period of record, such that the effect of short-time and irregular movements is largely smoothed out. The use of smooth lines drawn by eye with the aid of French curves would appear to be satisfactory were it not for the fact that the unguided eye has the tendency, like mathematically fitted curves and arithmetic averages, to give undue weight to extreme items. If, therefore, a guide for the eye could be determined the matter would be largely solved.

A moving arithmetic average is obviously unsuitable for this purpose on account of its susceptibility to the influence of extreme items. A moving median might seem to be more satisfactory. It is a peculiarity of the moving median, however, to run along at a constant level for a considerable period of time and then suddenly rise or fall to a new level. It does not possess the smoothness of the arithmetic average. The type of moving average that we have found to be particularly suitable for this purpose may be regarded as a compromise between the arithmetic average and the median, a device which may be called the broadened moving median. It is determined as follows:

Suppose the number of items to be included in each moving average is seven, for example. The first seven items are arranged in order of magnitude and the arithmetic average of the central three is obtained. Then, dropping the first item and adding the eighth, these seven items are similarly arranged in order of magnitude, and the arithmetic average of the central three is found. And so on. The procedure is quite similar to that involved in finding the ordinary moving arithmetic average, except that instead of averaging all the items in each moving group the four extremes are excluded. The averages thus determined are centered and plotted on the same charts as the link relatives. It is evident that this type of moving average partakes of the smoothness of movement characteristic of the moving arithmetic mean, and possesses as well the freedom from distortion by extreme items characteristic of the moving median.

With this curve plotted as a guide, the remaining irregularity is eliminated by drawing in a smooth curve by means of French curves. It is true of course that the moving average does not extend to either end of the series, but the smooth curve may with little risk be extended as far as the link relatives are available. This last step calls for the exercise of some personal judgment, since it is inadvisable to follow too faithfully the minor movements of the link-relative curves. The latter fluctuations are due to a combination of seasonal, cyclical, and irregular

variations and must naturally be scrutinized with care. On the whole, however, it is felt that the resulting smooth curve reduces to as small a degree as is feasible the effect of individual judgment.

As to the period selected for the moving average, seven years has been found to be quite satisfactory, since it is of ample length to reduce to a minimum the effect of the cycle, and at the same time is short enough to reproduce the really significant movements of the link relatives. In the case of series which are so short that the use of a seven-year period would sacrifice too great a portion of the data, an average of three out of five items has been used with success.

The final steps in this method consist in reading from the twelve smooth curves found as just described a point for each year; and developing a separate set of seasonal indices for each year from these points by the usual procedure of the link-relative method. It may be noted that in practice where the curves are not moving rapidly from one year to the next it has been found sufficiently accurate to compute seasonal indices at intervals of two or three years, and to determine indices for the intervening years by straight line interpolation. In keeping current computations up to date it is advisable to make revisions and determine new seasonal factors every year.

Mr. Flinn felt that the determination of seasonals was, however, still in the experimental stage, and that the methods which he had been using might be improved.

The fourth address was by Mr. Victor S. von Szeliski, consulting statistician of 44 Broad Street, New York City. He called attention to the fact that almost all of the premises concerning seasonal variations remain to be demonstrated, and can at best be considered nothing but approximations to the truth. He suggested, as a method for demonstrating the comparative merits of different methods of measuring seasonal variations that empirical seasonal variations be superimposed upon a known smooth curve, such as the sine curve, and that the method then be applied to the combination. The best method should give the most accurate reproduction of the original seasonal that was added. Mr. von Szeliski said that he had tested by this method the link-relative method, the moving average method, and the ratio to secular trend method, and found that each gave a considerable error. In his opinion, the link-relative method and the secular trend method both contained the inherent weakness that they failed to eliminate extraneous fluctuations before beginning to measure the seasonal fluctuation. While the moving average method was more logical, it did not eliminate the extraneous movements with sufficient accuracy. He stated that, from a reading of the literature on seasonal variation, it would seem that most of the writers, with the exception of W. I. King, had entirely missed the point that it was essential to eliminate the cycle before beginning to measure the seasonal. Mr. von Szeliski outlined briefly a method which he expects to present more fully at a later time, namely, a means of approximating the cycle by aid of a second-degree parabola. He stated that, by this method, he had been able to reduce the error in the seasonal, as recomputed from the mathematical time series that he had constructed, to about one-tenth of the error present when the other methods were used. He is now engaged in developing his method for use with the changing seasonal.

The discussion was begun by Dr. Willford I. King of the National Bureau of Economic Research. He stated that, so far as he could learn, almost everyone who had investigated the seasonal fluctuations in historical variables by a method capable of detecting changes in these seasonal variations, had found that such changes were present. On the basis, then, of such limited experience as is available, it appears that the constant seasonal is the exception, rather than the rule, and, in view of such evidence, it is extremely dangerous to assume that the seasonal is unchanging, unless there is positive evidence establishing the fact. He contended that radical changes occur so frequently, that no method can pretend to be generally applicable that

provides only for measuring an average seasonal over a long period of time. Mr. King expressed the view that, in the case of most historical variables, it is entirely impossible to measure the actual seasonal variation in any year, because the seasonal is continually fluctuating on account of changes in climatic conditions, fashions, and matters of industrial technique. The most then that we can hope to accomplish is to approximate the normal seasonal for any given date. When such a normal seasonal is extracted, the residual figures will still contain a considerable seasonal element. He felt that, in order to measure this normal seasonal, logic compels the elimination, in so far as feasible, of all extraneous movements before beginning to measure the seasonal; in other words, both the trend and the cycle should be disposed of at the outset to the fullest extent possible. In his view the link-relative method is weak, in that it fails to get rid of these two forces at the start. In cases where the seasonal variations are large, the cyclical variations small, and the trend not very steep, almost any method will give satisfactory results. The real test of the merits of a method comes, however, when it is called upon to determine the seasonal in a case like that of pig-iron production, in which the cyclical variations are very large and abrupt, and in which the seasonal variation is extremely small.

Mr. King granted without hesitancy that the elimination of the cycle and trend by means of a free-hand curve placed by inspection was an imperfect method and one that was distinctly lacking in mathematical nicety. He further agreed that, in the case of pig-iron production, the preliminary seasonals obtained by dividing the original items by a cycle curve of this sort were widely scattered, and that, therefore, the moving median represented merely a crude approximation to the central tendency. He also admitted that the procedure must be imperfect, inasmuch as the ratios found could not be reproduced in exactitude from the final estimates. He agreed heartily with Mr. Crum in the view that, other things being equal, purely mechanical methods are to be preferred but stressed the necessity of ever being on one's guard against those deceptive appearances of accuracy which often serve to screen the grossest errors. One cannot be too certain of the quality of a formula if he is to make it his master.

Mr. King was strongly of the opinion that the mechanical methods proposed by Mr. Crum, while they might give the illusion of precision, really contained errors so large as to overshadow completely those present in the figures resulting from the method devised by Mr. King [1] and presented in the September, 1924, JOURNAL. Since cyclical fluctuations are not eliminated at the outset in computing link relatives, the errors therein will commonly be much greater than the errors in the ratios of the actual items to the assumed cyclical curve. This will be true even if the preliminary cycle curve is drawn with but a modicum of skill, for anyone having the faintest conception of the cycle can get rid of most of the cyclical movement without difficulty. While granting that the necessity of using personal discretion is an undesirable factor, Mr. King stated that he had found by actual test that it was not at all difficult to train a statistical clerk in a few hours to carry through the necessary operations in his method, without introducing undue errors. He scouted the idea that the course of the changes in the seasonal variations shown in the case of pig-iron production was merely a reflection of the way in which the original curve was drawn.

It was his view, however, that, if an easy and accurate mechanical method could be

[1] The idea that the normal seasonals for the respective months vary along different trends was originated by Dr. Edgar Sydenstricker, and a method for dealing with this changing seasonal was worked out under his direction by Mr. Rollo Britten. By revealing the imperfections in the ordinary methods of measuring seasonals, Dr. Frederick R. Macaulay contributed materially to the development of the method in the form presented.

devised for placing this cycle curve, such a method should be adopted without hesitation, but he felt that no such method has as yet been presented to the public. He hoped that the procedure proposed by Mr. von Szeliski might fill this need, but he feared that it would call for such a lengthy process of computation as to be prohibitive in cost. Since skilled mathematicians are rare, any method which is to be widely serviceable ought not to require the use of complicated mathematics, and since most concerns cannot afford to devote an undue amount of time to the elimination of seasonals, computation should be reduced to the minimum necessary to give dependable results. Mr. King felt that, in most instances, the final results would be practically the same, if determined upon the basis of a free-hand curve hastily drawn or if calculated from a cycle curve located by mathematical methods of the highest precision known. He doubted that, as a matter of fact, the mathematical devices heretofore advanced would really give as accurate a record of the cycle as would a free-hand curve—provided that the person drawing the free-hand cycle curve first took the precaution of inspecting carefully the original curve, and, after making notes of the maximum probable size of the seasonal variations for each of the months, so drew the preliminary free-hand curve as to keep within this maximum distance from the original data for the given month.

While Mr. King agreed that, in the case of pig-iron production, the scatter of the ratios first computed was so great as to make the moving median but a rough approximation to the true course of the seasonal for any given month, he felt that the scatter obtained by this method, while too large, was at least very much smaller than it would be either if link relatives were used, or if deviations were taken from the trend of the original data. While the scatter would be less in the case of percentage deviations from the moving average than it would be by either the Persons or the Falkner method, it would still be materially greater than when taken from a free-hand cycle curve even if the latter were crudely drawn, for the moving average has a tendency to introduce unnecessary errors by cutting corners.

In Mr. King's opinion, Mr. Crum's assumption that the problem is one of regression is erroneous. It seemed to him that the shape of the monthly seasonal curves presented on page 309 of this JOURNAL for September, 1924, was sufficient answer to Mr. Crum's proposal to approximate these curves by straight lines, or even by parabolas. While Mr. King was very doubtful that the minor fluctuations in these lines were significant, he felt that there was no question about the existence of the major tendencies shown, and that the monthly curves are very irregular in their courses. He stated that, in his opinion, the validity of the shape of these curves had been substantiated to a notable degree by the studies of Dr. Harry Jerome, who had carried the curves back to an earlier date, and had found that the course of the earlier movements gave support to those presented in the charts in at least nine out of the twelve months. Mr. King therefore adhered to the view that the method which he had presented would, in general, give results sufficiently accurate for all practical purposes, even in cases in which the measurement of the seasonal was extremely difficult.

However, he believed that the method was susceptible of distinct improvement. For example, the device suggested by Mr. Flinn of arraying the ratios and then averaging the three middle items was probably better in many cases than taking the median. Improvements of this nature should be tested and adopted if they prove successful, but it is highly desirable that the method be kept simple enough to be understood by persons with little mathematical training, and that the computation be not extended, unless it can be demonstrated that, by so doing, distinctly more accurate results can be obtained.

The discussion was continued by Mr. Raymond B. Prescott, consulting statistician of 44 Broad Street, New York City. Mr. Prescott detailed some of his experiences in dealing with automobile statistics. He stated that the seasonal fluctuations in automobile production curves change radically from year to year, and that the changes are extremely erratic. In his view, the reason for such irregularity is that production is dominated by the actions of a few executives and that it varies according to their decisions, which are not governed by any recurrent seasonal laws. He has found, however, that the case is very different with the volume of retail sales, since this volume depends upon the wishes of millions of people, and hence attains statistical regularity. As a result, the seasonal variations in retail sales can be measured with considerable accuracy.

Mr. Jerome called the attention of those present to the fact that Mr. Crum's view that there is sometimes a cycle in the seasonal was confirmed by some of the data pertaining to migration which he had been studying. He ascribed this to the fact that the seasonal fluctuation is affected by the same force that gives rise to the cyclical movement.

The discussion was closed by Mr. Frederick R. Macaulay of the National Bureau of Economic Research. He stated that, in general, he favored the use of a method akin to that presented by Mr. King in the September JOURNAL, and that he did not endorse the method ascribed to him by some of the earlier speakers, in which the original items were divided by the moving average. He believed that the moving average fails to locate the preliminary cycle curve with sufficient accuracy but that it nevertheless furnishes a good preliminary basis for locating it, and felt that the method presented by Mr. King would be improved upon, if the original cycle curve were so drawn as to make it include in each year approximately the same area underneath it that lay beneath the moving average curve for the same year. He favored, however, the use of a smooth empirical free-hand curve adjusted to this area.

Mr. Arne Fisher of the Western Union Company expressed the view that the cycle curve could best be fitted by a modification of the Fourier series. Mr. Macaulay, however, was of the opinion that such a mathematical fitting of a cycle curve covering a long period of years would be an extremely time-consuming process, and that, in the end, it would be less accurate than the free-hand curve fitted by the method that he had just suggested. If so, the work invested would represent rather a concession to mathematical elegance than a contribution to our knowledge either of the course of the cycle curve or the extent of the seasonal variation.

At the close of the meeting a resolution was offered and adopted that the Executive Committee of the American Statistical Association be requested to appoint a committee under the chairmanship of W. I. King, the purpose of which would be to attempt to coördinate the various methods of analyzing seasonal variations. The meeting then adjourned.

United States Bureau of Labor Statistics.—The field investigation into the extent of phosphorus necrosis as an occupational disease in the manufacture of certain kinds of fireworks is complete and the preparation of the report well under way.

The Bureau has been recently engaged in an investigation of radium necrosis as an occupational disease in the manufacture of luminous paints and the application of these paints to clock dials and watch dials, etc. The field work is finished and the results will be published in the near future.

Dr. R. R. Kuczynski's remarkable bulletin on "Post-war Labor Conditions in Germany" has been issued by the Bureau as Bulletin No. 380.

Under the title of "Coal Situation in Illinois" the Bureau has recast the bituminous

coal statistics as furnished by that state. The significant conclusions reached by the Bureau's method of handling the figures are that the so-called wagon mines or snowbird mines are not worth considering in the real bituminous coal problem, that while there are 1032 operating coal mines in the state only 338 of these are really shipping mines and worthy of consideration. Of these 338 mines 84 or 24.9 per cent of the mines had they operated 300 days in the year could have produced 10 per cent more coal than they all produced by employing 51.5 per cent of the workers actually employed. The statement is made: "This means that 254 of the 338 principal mines of the state represent an unnecessary expenditure of money so far as the capital investment in the mines themselves is concerned, that they simply prevent an adequate number of mines from producing an adequate amount of coal on a reasonably efficient basis, that the railroads must distribute cars to 254 unnecessary mines in a single state with all the capital invested in cars that this implies." It is shown that only 27 mines operated 200 days or over, and these were mines either owned by the railroads or selling most of their production to railroads.

The field investigation of tableware potteries, which is taking a much broader scope than a mere wages investigation, is well under way. Mr. Jesse C. Bowen, the Chief Statistician of the Bureau, is spending some time in England and France in an effort to secure corresponding information in those countries on the subject.

In an effort to determine if possible the amount of actual building being done, the Bureau has devised a building construction index built up of four factors—the money value of new work as indicated by building permits, the cost of building material, the wages of labor in the building trades, and the relation between material costs and labor costs in the completed structure. This index is compared with the growth of population. The index for the country as a whole is published in the July *Monthly Labor Review*. A similar index by cities is in preparation.

Institute for Biological Research at the Johns Hopkins University.—The Rockefeller Foundation, through its Division of Studies, has appropriated a substantial sum to aid in the establishment, and support for a period of five years, of an Institute for Biological Research at the Johns Hopkins University. This Institute will be closely affiliated with the Schools of Medicine and of Hygiene and Public Health of the University. Doctor Raymond Pearl has been appointed director. He will retain a connection with the Department of Biometry and Vital Statistics of the School of Hygiene, as research professor in this subject, and will continue as professor of biology in the Medical School. Dr. Lowell J. Reed becomes professor of biometry and vital statistics and head of the department in the School of Hygiene.

At the outstart the Institute for Biological Research will occupy one floor of the New Hunterian Laboratory of the Medical School. The whole time of the staff, which is now being chosen, will be devoted to research on general problems of biology, but with especial attention to the biology of life duration and its control, and to the experimental study of the population problem.

Bureau of Statistical Research at the University of Denver.—For purposes of analysis of economic and social welfare, the Rocky Mountain Region probably offers as clear a story of related elements as is available. Denver is a cosmopolitan center with an extensive hinterland, reaching into eight or more states. This has been a large factor in the organization of the Bureau of Statistical Research of the University of Denver, School of Commerce, Accounts, and Finance.

Another objective has had almost equal weight. There are in training in the University, a group of men and women who are natives of this region and whose

influence in the planning of its future will depend largely upon their training in objective and method. For the students, this region is made a laboratory.

The Bureau was established in June, 1924, by Dr. G. A. Warfield, Dean of the School of Commerce, with Prof. John H. Cover, formerly of Columbia University, as Director. In addition to the collection and analysis of fundamental series, a complete survey of the region is under way, covering social, economic, physical, and governmental aspects. A monthly publication, the *University of Denver Business Review*, presents analyses of individual series and has already covered the following: Bank Clearings, Building Permits, Coal Production, Cattle and Sheep Marketing, Gas and Electricity Consumption, and Pig Iron Production. The trend, seasonal variation, and cycle are derived in each case and comparison made of the regional situation with the country as a whole.

Two fellowships in economic research are awarded annually, the fellows being required to write their theses in the general field. Students in statistical method and in advanced courses in business cycles, forecasting, and research seminar contribute toward the program of studies. Mr. F. L. Carmichael, formerly Fellow in Mathematics, University of Princeton, and Assistant Professor of Mathematics in the University of Alabama, has just been appointed Associate Director.

Tenth Anniversary Meeting of the Casualty Actuarial Society.—At the twenty-third regular meeting of the Casualty Actuarial Society held at the Hotel Pennsylvania, New York City, on November 20, 1924, the tenth anniversary of the Society was celebrated. The transactions of the meeting are reported in the Proceedings of the Society, Volume XI, Part I, November 20, 1924. In addition to technical papers, the Society heard addresses from the President, Mr. William Leslie, on "Casualty Problems from the Public Viewpoint," an address by Dr. I. M. Rubinow on "Origin of the Casualty Actuarial Society," one by Mr. J. D. Craig on "Relation of the Casualty Actuarial Society to Other Scientific Organizations" and a historical paper by Mr. B. D. Flynn, "Review of the Society's First Ten Years."

The educational program of the Society was discussed by Messrs. Fondiller and Kopf. An interesting section of this issue of the proceedings is a series of histories of the principal actuarial and statistical societies of the world. The articles on the American Statistical Association and the Royal Statistical Society were contributed by Mr. E. W. Kopf.

The objects of the Society are the cultivation of actuarial and statistical science in relation to the problems of casualty and social insurance. During the past ten years, the Society has rendered noteworthy services, especially in the scientific development of workmen's compensation insurance. At the present time, the Society is looking forward to another decade of constructive service to the insurance business. Its educational program is being revised in the light of the experience of ten years in prescribing a syllabus of study and in conducting examinations for proficiency in the actuarial and statistical sciences supporting the casualty insurance business.

A Graduate School of Business is to be established at Leland Stanford University, California. Willard E. Hotchkiss of the National Industrial Federation of Clothing Manufacturers has accepted the appointment as Dean.

Mr. C. W. Cousins, M.A., who was Director of Census and Statistics for the Union of South Africa from 1917, has been appointed Secretary for Labour in the Union. Prof. J. E. Holloway, B.A., D.Sc. (Econ.) of the Transvaal University College, Pretoria, has been appointed his successor in the Directorship of Census and Statistics.

Dr. Raymond Pearl of the Johns Hopkins University has been elected an honorary fellow of the Royal Statistical Society of London.

The United States Commissioner of Labor Statistics, Ethelbert Stewart, plans to attend the conference of the International Statistical Institute in September.

At the invitation of the Health Section of the League of Nations, Dr. William H. Davis, Chief Statistician for Vital Statistics, in the Bureau of the Census, has joined the tour of medical statisticians, who will meet at Copenhagen July 28, and visit Stockholm, Oslo, London, Geneva, and probably a few other European capitals to study the statistical methods of the various countries. Invitations have been sent to representatives of 11 countries: Finland, Norway, Sweden, Denmark, Netherlands, Belgium, France, Switzerland, England, Scotland, and the United States. The tour will last about a month.

The eighty-seventh annual meeting of the American Statistical Association will be held at New York City on December 29–31, 1925. The headquarters of the meeting will be at the Pennsylvania Hotel. The American Economic Association, the American Sociological Society, and the American Political Science Association will meet at the same time and place. Professor Roswell C. McCrea of Columbia University has been made chairman of the Joint Committee on Arrangements, and Professor Frank A. Ross of Columbia University has been appointed representative of the American Statistical Association on this committee.

Professor Walter F. Willcox has been elected by the Board of Directors as the representative of the American Statistical Association on the Board of Supervising Editors of the American Year Book for the year 1925.

Professor Irving Fisher was elected as a member of the Committee on Standardization of Graphics at the Board of Directors meeting July 16, 1925.

ADDITIONAL COMMITTEE APPOINTMENTS, 1925

Committee on Fellows:

	Term Expires
Warren M. Persons	1925
Edmund E. Day	1926
Joseph A. Hill	1927
Raymond Pearl	1928
Louis I. Dublin	1929

Committee on Classification of Statistical Workers in Government Service—Edwin F. Gay (Chairman), Kate H. Claghorn, Allan H. Willett.

Committee on Handbook of the Social Sciences—Mary Van Kleeck (Chairman), Wm. F. Ogburn, R. H. Coats.

Joint Committee of Arrangements for the Annual Meeting, December, 1925—Frank A. Ross. (Roswell C. McCrea of the Economics Association is Chairman of this Committee).

Committee on Nominations—Horace Secrist (Chairman), W. Randolph Burgess, M. O. Lorenz.

MEMBERS ELECTED SINCE JUNE, 1925

Anderson, Arnold T., Security Trust and Savings Bank, Los Angeles, Calif.
Angus, William Newton, (Construction Statistics), 161 W. 105th St., New York City.
Barbour, Philip L., Security Trust and Savings Bank, Los Angeles, Calif.
Beaver, Mrs. Amba O., *New York Times* Annalist, New York City.
Benner, Claude L., Institute of Economics, Washington, D. C.
Berry, Charles E., Merchants National Bank, Los Angeles, Calif.
Black, Albert G., School of Business, University of Minnesota, Minneapolis, Minn.
Bodwell, Henry M., Western Electric Co., 195 Broadway, New York City.
Boettler, Herbert F., First National Bank in St. Louis, St. Louis, Mo.
Bouton, Gladwin, Harvard University Committee on Economic Research, Cambridge, Mass.
Brown, Donaldson, General Motors Corporation, 224 W. 57th St., New York City.
Bryan, Barnabas, Standard Oil Co. of N. J., 26 Broadway, New York City.
Bunke, John R. D., Weber-Bunke-Lange Coal Co., 270 W. 96th St., New York City.
Chao, Shao Ting, St. John's University, Shanghai, China.
Clapp, Raymond F., Associate Director, Welfare Federation of Cleveland, 513 Electric Bldg., Cleveland, Ohio.
Craig, David R., University of Pittsburgh, Pittsburgh, Pa.
Craig, Harold E., First National Bank, Los Angeles, Calif.
Cromwell, Alexander, The American Appraisal Co., 904 Van Nuys Bldg., Los Angeles, Calif.
Eisenberg, Martin F., Eisenberg, Baker, Copeland & Associates, Asheville, N. C.
Elliott, Foster F., Bureau of Agricultural Economics, Washington, D. C.
Fader, Edmund F., Eberle & Riggleman Economic Service, 1121 Van Nuys Bldg., Los Angeles, Calif.
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Houghton, Ruth, American Telephone and Telegraph Co., 195 Broadway, New York City.
Hulse, Edward Pierce, American Type Founders Co., Jersey City, New Jersey.
Huntley, Kate E., National Bureau of Economic Research, 474 W. 24th St., New York City.
Johnston, Douglas T., Irving Bank, Columbia Trust Co., 60 Broadway, New York City.
Jones, Marvin Z., U. S. Dept. of Labor, Bureau of Labor Statistics, Washington, D. C.
Kayser, Charles, Federal Reserve Bank of New York, New York City.
Killam, Maude, American Telephone & Telegraph Co., 195 Broadway, New York City.
Knoles, Fred A., 4316 Kenwood Ave., Los Angeles, Calif. (Los Angeles Police Dept.)
Knote, Frederick C., Federal Reserve Bank, Reports Dept., New York City.
Kuenzel, Myra W., Ohio State University, Columbus, Ohio.
Latour, Casper C., Scholle Bros., 5 Nassau St., New York City.

Lauder, Mildred M., Sales Record Bureau, American Radiator Co., 1807 Elmwood Ave., Buffalo, New York.
Lee, Edward B., Electric Bond & Share Co., New York City.
Lehmann, Ferdinand C., All America Cables, 89 Broad St., New York City.
Markey, John F., University of Minnesota, Minneapolis, Minn.
Metzger, Clarence B., The Edward A. Woods Co., Frick Bldg., Pittsburgh, Pa.
Millward, Albert E., Dept. of the Interior, Ottawa, Canada.
Moody, John, Jr., Moody's Investors Service, New York City.
Mortara, Giorgio, Professor of Statistics, Royal University of Milano, Milan, Italy.
Nance, Miss Mollie, Census Bureau, Washington, D. C.
Niceley, J. L., Eberle & Riggleman, 1121 Van Nuys Bldg., Los Angeles, Calif.
O'Connor, John Jay, Finance Dept., Chamber of Commerce of the United States, Washington, D. C.
Omohundro, Edgar H., 121 Chestnut St., Philadelphia, Pa.
Parker, Coralie, Johnson Hall, Columbia University, New York City.
Perry, F. Gardiner, Babson's Statistical Organization, Wellesley Hills, Mass.
Peterson, Ruth E. K., United States Tariff Commission, Washington, D. C.
Pieper, Karl V., Commodity Statistics, Inc., Cambridge, Mass.
Pierce, F. L., National Carbon Co., Inc., 30 E. 42nd St., New York City.
Pollak, Inez, 260 Riverside Drive., New York City.
Prentiss, Mary Alden, 2880 Broadway, New York City.
Rogers, Osgood S., American Telephone and Telegraph Co., 195 Broadway, New York City.
Root, H. M., Comptroller, Southern Baking Co., Atlanta, Ga.
Ross, Thurston H., Dept. of Industrial Engineering, Univ. of Southern California, Los Angeles, Calif.
Ruger, Henry A., Teachers College, Columbia University, New York City.
Salant, Aaron B., 1155 Park Ave., New York City.
Schmidt, Dr. John E., 1195 Dean St., New York City.
Seaton, G. Leland, Illinois Bell Telephone Co., Room 1301, 212 Washington St., Chicago, Ill.
Sellew, George Tucker, Knox College, Galesburg, Ill.
Sevier, Frederick, Port of New York Authority, 11 Broadway, New York City.
Shewhart, Walter A., Bell Telephone Laboratories, Inc., 463 West St., New York City.
Snedecor, George W., Iowa State College, Ames, Iowa.
Sorenson, Herbert, Northwestern University, Evanston, Ill.
Soule, George, Labor Bureau, Inc., 2 W. 43rd St., New York City.
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Stern, Sidney J., 8 W. 40th St., New York City.
Stevenot, Charles J., Charles Stevenot & Co., Inc., New York City.
Stone, Thomas E. Jr., White, Weld & Co., 14 Wall St., New York City.
Thiem, Robert S., Mathieson Alkali Works, Inc., 250 Park Ave., New York City.
Tomkins, Esther G., 247 Fifth Ave., New York City.
Vial, Edmund E., College of Agriculture, Ithaca, New York.
Voegelin, Frederick E., Case, Pomeroy & Co., Inc., New York City.
Ward, H. H., Labour Dept., Ottawa, Canada.
Warde, Ethel M., Emery School, Washington, D. C.
Welch, Philip Albro, Massachusetts Institute of Technology, Cambridge, Mass.
Welling, William Corcoran, Connecticut State Dept. of Health, Hartford, Conn.
Wiecking, Ernst H., Bureau of Agricultural Economics, Washington, D. C.

Wilcox, Lloyd, Dept. of Sociology, Ohio State University, Columbus, Ohio.
Wilder, Francis S., Dept. of Sociology, Chapel Hill, N. C.
Willard, Leigh, 52 Vanderbilt Ave., New York City.
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REVIEWS

Social Consequences of Business Cycles, by Maurice B. Hexter. Boston: Houghton Mifflin Company. 1925. xiv, 206 pp. (Number 7, Publications of the Pollak Foundation for Economic Research.)

It is difficult to understand why Dr. Maurice B. Hexter called this collection of intricate statistical analyses and theorizing "Social *Consequences* of Business Cycles," for the point to his whole argument is the effect of social forces, particularly cyclical fluctuations in births and deaths, in bringing about and influencing the course of business cycles.

The argument depends largely on the results of correlating series of social statistics with series of economic statistics for the Boston district over the period 1903–1916. The data comprised are, on the one hand, birth-rates, still-birth-rates, death-rates, marriages, and divorce cases filed; on the other hand, wholesale prices and employment. The correlations are usually computed for synchronous items and for a lag and lead in the social series of from one to twenty-five months.

In regard to births, the author concludes that fluctuations in conceptions precede like fluctuations in wholesale prices by eight months, and that fluctuations in births precede like fluctuations in unemployment by seventeen months. Still-birth-rate cycles precede wholesale price cycles by two months. Death-rate cycles precede wholesale price cycles by seventeen months and unemployment cycles (negatively) by ten months. Marriages lag one month behind wholesale price cycles, and divorce rates lag twenty-four months (negatively) behind wholesale price cycles.

Dr. Hexter's generalizations in regard to the lags and leads of these social series have very little scientific basis. The use of correlation coefficients in determining the lag or lead in cyclical time series is open to serious difficulties, especially where the cycle is irregular in length and in amplitude. For, in a cyclical series, an apparent lead in one series may actually be a lag from the preceding cycle, and *vice versa*. A further complicating factor is the tendency of a lag or lead to change its length from one cycle to another. In short time series, such as Dr. Hexter uses, there is no scientific way of determining whether there is a relatively short lead or a relatively long lag in their relationships to the business cycle. Judgment depends ultimately on the *a priori* probability of the occurrence. If correlation coefficients are worked out for a long enough lag and lead, the size of the coefficient is a clue to the validity of assuming a lag or a lead, and gives the average length of the assumed lag or lead. But, since chance deviations may be heavily weighted in the correlation coefficient, it is not possible to assume that *small* numerical differences between coefficients can be a sufficient criterion for determining the lag or lead. In practically every series Dr. Hexter has correlated, there is mathematical evidence of both lag and lead. For instance, in birth-rates and employment through January, 1916, a lag of 17 months gives a maximum positive coefficient of +.292, and a lead of 17 months gives a maxi-

mum negative coefficient of −.302, yet Dr. Hexter assumes the lead to be the valid relationship. Likewise, in correlating still-births and wholesale prices, a lead of 2 months gives +.348 against +.283 for a lag of 24 months for one sub-period; and for the other sub-periods, +.176 lead against +.187 lag, and +.158 lead against +.155 lag. Death-rates correlated with wholesale prices give leads of +.613, +.416, and +.392 against lags of +.447, +.263, and +.213 respectively. These numerical differences in the coefficients of correlation are, in most cases, far too small to give any real evidence of a consistent lead or lag in the social phenomena, yet they are apparently the foundations of Dr. Hexter's thesis.

But the author is not consistent in accepting numerical differences between coefficients of correlation as his criterion of lag or lead, for, in correlating marriage rates and wholesale prices, he claims a positive correlation with marriages lagging one month. Now a lag of one month in marriages gives coefficients of +.469, +.223, and +.154 for the three sub-periods of the study, whereas a lead of 28 months gives coefficients of −.442, −.280, and −.252 for the corresponding sub-periods. It would seem that when it suits Dr. Hexter's bias, he assumes a lag or a lead in his social series without much reference to the actual correlation coefficients.

A further point arises in regard to the alleged lead of conceptions and births over wholesale prices and employment. Yule showed that the birth-rate tends to lag more than two years behind the marriage rate, and Hooker found a lag of more than half a year in the marriage rate behind trade and employment—all of which suggest a much longer lag than Dr. Hexter has tested in his correlations of births and economic series.

For purposes of measuring the relation of social phenomena to the business cycle, the period used in this study, 1903–1916, is very short. Dr. Hexter calls attention to the number of *pairs* of items involved in the correlations, but it should be remembered that in tracing cyclical relationships of this sort it is perhaps more important to have a large number of cycles than a large number of pairs of items for a few cycles. It is interesting and important to note the month by month comparisons, but it is unfortunate that so few cycles were included in the series.

Dr. Hexter's conclusions, arising from this statistical "evidence" of a lead in certain social series over the economic series, are more interesting than convincing. Cyclical fluctuations in birth- and death-rates, occurring apparently through forces of their own, he claims, give an explanation of the origins of business cycles. "Let us see how this may occur. A man leaves a sick child at home. He goes to a meeting of the directors of his corporation. A project to expand and capture new markets is suggested. The man cannot devote his full thought to the discussion: his mind is in the sick room at home. He proposes that the decision go over until the next meeting. His emotions are infectious. He carries his point. . . . Still another is told on his return home that a child can be expected in due course. He is thrilled. He must prepare financially for its coming. He quickens his efforts. He wants to rise higher in his field of endeavor. He works harder and better. He achieves more. His results reflect his redoubled efforts.

"Such instances of the actual effect of deaths and births upon men are not unusual. Biographies and autobiographies are full of them. The stage reflects them. Can there be any doubt that there is a close connection between these fluctuations in birth-rate and death-rate and fluctuations in business enterprise?" (Page 174.)

All this adds little of value to our knowledge either of the facts of the business cycle or its social consequences. There are so many limitations to the use of statistics as a tool in developing a scientific sociology, that it will require a much more rigorously objective treatment than Dr. Hexter has given to bring sociological investigation even to the threshold of science.

DOROTHY SWAINE THOMAS

New York City

Purchasing Power of the Consumer: A Statistical Index, by W. A. Berridge, E. A. Winslow, and R. A. Flynn. New York: A. W. Shaw Company. 1925. 318 pp.

Incomes of consumers are an important factor in the effective demand for commodities, and in order to encourage the development of a quantitative measure of purchasing power the J. Walter Thompson Company has awarded three prizes for essays on the subject, "A Statistical Index of the Purchasing Power of the Consumer in the United States." The three prize-winning essays, which have been published in a single volume, offer a useful compendium of information regarding the sources and uses of the statistics that indicate purchasing power of the consumer.

Each of the three authors has a distinctive approach to the problem, but all three have apparently kept in mind the fact that the essays were written for an advertising and selling organization and have endeavored to show how the particular statistics under discussion could be used in determining advertising and selling policy. The essays give a statistical approach to the problem of "educating" the public in what to buy and set forth certain principles, supported by statistics, which should guide those who would appeal to the tastes of the various groups of society in order to reach their pocket-books. Nevertheless, the first two essays do provide ideas and working tools for economists and statisticians who are more directly concerned with the problems of wealth distribution and consumption or are interested in methods of studying these problems.

In the essay winning the first prize, Mr. Berridge, who has previously made many important contributions in the field of employment statistics, presents a series of index numbers of the incomes of factory workers. Miss Winslow, winner of the second prize, has made a careful analysis of various budget studies with the view of showing how budget information might be used to indicate the character of goods that are most likely to be purchased by different groups of consumers or by consumers in different localities. Mr. Flynn, in the third essay, catalogues and to a certain extent explains the various statistics collected by an enterprising state department of labor.

In computing his new index of earnings of factory workers, Mr. Berridge has used pay-roll data and methods similar to those previously employed by him in computing indexes of factory employment for certain of his own employment studies, and for the Harvard Review of Economic Statistics, the Brown Bureau of Business Research, and the Federal Reserve Board. The purpose of the index is to show "primarily the time fluctuations and incidentally the geographical distribution of incomes received by factory workers in the United States" in the post-war period.

The reasons given for constructing such an index as indicative of the purchasing power of consumers are that factory wage earners are more numerous than any other single group of income recipients, that they spend such a large part of their incomes for consumers' goods, and that the volume of such earnings is very sensitive to current changes in business. Furthermore, statistical data for the computation of an index of this character are available over a long period of time and are more reliable than any other evidence of the buying power of consumers. Some of the above reasons, of course, might vitiate the use of such an index in certain communities. For example, in agricultural sections it would be misleading and useless, because incomes of farmers frequently show changes widely different from those of factory workers. It has been shown that fluctuations in industrial activity and changes in agricultural conditions have little similarity. Moreover, according to census figures, the number of persons gainfully employed in agricultural pursuits is almost as large as those engaged in manufacturing; the rural population of the country, which is almost entirely agricultural, is nearly one-half of the total; and many urban centers are directly dependent upon agricultural communities. Mining, trade, and transportation represent other important phases of activity, changes in which are only indirectly reflected by fluctuations in factory pay-rolls.

In computing this index, Mr. Berridge used figures collected by the United States Bureau of Labor Statistics, the New York, Massachusetts, Illinois, and Wisconsin departments of labor, and the Federal Reserve Bank of Philadelphia, in order to secure as wide industrial and geographical representation as possible. Industry indexes were computed on a 1919–1922 base from the United States and New York series. A weighted arithmetic average of these industry relatives gave a general index with which corresponding indexes for Wisconsin, Illinois, Massachusetts, and the Third Federal Reserve District were incorporated at different points of time. A special feature is made of the industry indexes, which Mr. Berridge suggests might be used to indicate conditions in sections of the country where particular industries are important. These indexes, however, are made up entirely of data from the United States Bureau of Labor Statistics, and the New York Department of Labor. The statistics from the Federal Bureau prior to July, 1922, were frequently inadequate and covered only twelve industries. If Mr. Berridge had incorporated data from other states into his industry indexes and had made adjustments in accordance with the trend of census figures in 1919 and 1921, as he did in the Federal Reserve Board's employment indexes, the representativeness of his indexes of pay-rolls by industries would have been increased. A geographical index for New England is computed by combining

various indexes covering the entire country for certain industries, weighted according to their relative importance in New England. A comparison of the resulting index of factory-workers' incomes for New England with an index of business activity for that section, computed by the Federal Reserve Bank of Boston, does not show as close a correlation as one would expect.

The author compares the general index of purchasing power with certain other economic variables such as employment, wage rates, savings, advertising, retail sales, bank debits, and a business barometer. The comparisons are not entirely satisfactory, unless due consideration is given to the many limitations in all the indexes compared. In general it can be said that in presenting and explaining this new index, the author has made another useful contribution to the field of business statistics.

Whereas Mr. Berridge's index shows "trends in consumer demand at successive periods of time," Miss Winslow explains that her analysis of budget information is to show "the probable purchasing power of a certain number of persons living at a certain period of time under certain living conditions, as an aid to judging the market value of a particular community for a particular product." She lists and evaluates the various sources of budget data, and by the use of information from a broad collection of studies made in this and other countries she reaches certain generalizations regarding the buying habits of consumers. The distribution of expenditures among different purposes and among different kinds and grades of goods are shown to vary in accordance with size of income, size of family, composition of family, urban and rural conditions, geographical location, and types of community. Examples are given in considerable detail as to how this information might be used to indicate changes in demand for different products in particular types of communities with changes in income of consumers.

In general, expenditures for housing, fuel and light, and clothing show fairly fixed relations to total income, and this information "makes possible the rough calculation of the approximate purchasing power of consumers along these three lines from any statistics which might be available showing an average income for a specified locality." Information regarding "distribution of income into different groups and the distribution of population into families of different size" is needed to determine relative distribution of income among expenditures for food, sundries and savings. This prerequisite information, which is not supplied, makes difficult the application of these generalizations to particular problems. Other difficulties, which are recognized by the author, are that information to be derived from these studies is general and not readily applicable to specific products, that it is difficult to gauge statistically the changing character of communities, and also that such statistics as are available apply only to specific groups and therefore may be representative of only a small part of a particular locality. Miss Winslow concludes with suggestions as to the collection of statistics needed as a "basis for the more accurate construction of" an index of purchasing power.

Among the many statistics explained by Mr. Flynn in the third essay, the most useful as an index of the purchasing power of consumers is said to be the statistics "showing changes in the number of workers seeking employment for each 100

persons called for by employers through public employment offices." The prime reason given for the utility of these statistics is their timelinesss. The chart which is presented to show this ratio for New York state is not convincing as to its utility. Rather extreme seasonal fluctuations and occasional erratic movements make difficult any reasonable determination of trends. Another fault not shown, because data from only one state are presented, is that these ratios vary considerably in character and amplitude of movement in different states. Early in 1924 the Federal Reserve Bulletin contained a study in which experiments were made, with a fair degree of success, in adjusting these statistics for seasonal variations and differences in amplitude in order to make them more susceptible to interpretation. Mr. Flynn made no suggestion as to the advisability of such adjustments.

All of the essays point out the inadequacy and weakness of statistics now available and suggest programs for possible improvements and additions. Geographical data are particularly needed. Many charts and tables are presented with the essays providing information which can be applied by others to their particular problems.

Although the three essays were written without collaboration between the authors and treat of different data, they are so peculiarly related that they present a remarkably systematic approach to the problem of determining the purchasing power of consumers. Miss Winslow's conclusions regarding income expenditure used in connection with Mr. Berridge's index and Mr. Flynn's statistics, which indicate current conditions, should at a time and in a place provide a fairly accurate indication of consumer purchasing power. Any determination of policy must take into consideration not alone the total income of consumers, but also the class of consumer to be reached, the section of the country, the purpose in mind—to sell work shirts or to secure contributions for charity—the current trend of business, the season of the year, and other variables. And in the final procedure of evaluating individually the statistics given as answers to these questions and of determining their collective significance, considerable latitude remains for personal judgment. The essays, apparently without intent, make evident the futility of expecting to secure a single infallible index which will serve all purposes at all times and under all conditions. They do, however, indicate certain basic information which will be of assistance in reaching decisions.

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Federal Reserve System in Operation, by E. A. Goldenweiser. New York: McGraw-Hill Book Company. 1925. vi, 349 pp.

Several influences combine to make the position of the Federal Reserve System a difficult one. One is that it is an introduction of the central bank idea into a vast country where decentralized banking had prevailed since the days of Andrew Jackson. Centralization of reserves and of control is accordingly bound to bring friction and criticism even if conditions were propitious for such a change.

A second difficulty is that the law was passed late in 1913, and the system organized in 1914, but promptly and extensively modified to meet the situation created by the war. In a few years the war was over, the period of post-war inflation came with its disastrous aftermath, and the Federal Reserve management has been compelled to face a series of criticisms.

While these attacks have been numerous, there are two deserving of special comment. One came just after the business crisis of 1920, and was on the whole met successfully. A second one seems to be impending, less virulent in its nature, but perhaps fully as important. Mr. McFadden, Chairman of the Committee on Banking and Currency of the House of Representatives, announces his intention of presenting in Congress a bill for the modification of the Federal Reserve Act. Its purpose is, in general, to repeal certain amendments to the law that were adopted during the war on the assumptions that they were emergency measures, that the emergency has now passed, and that there is too much financial power now lodged in the hands of a relatively few men.

There is more involved in the proposal than appears on the surface and in the comments that have appeared regarding the proposed amendments. Concentration of function and power with Federal officials has been proceeding for years, and in some directions has doubtless gone too far and been too rapid. Centralizing forces are powerful, but there are many arguments in favor of a reversal of the movement. The defeat of the attempt to pass the Child Labor Amendment and President Coolidge's pleas that State and local governments assume more responsibility for the enforcement of law are illustrations.

This issue and others are involved in the criticisms of the Federal Reserve System, and what is needed are facts presented in a convenient form. These have been given and given well by Mr. Goldenweiser. About one-half of his volume is text and the remainder consists of appendices. The treatment may prove a little condensed for an introductory study by college classes, as the author intends, but it could be used as a text. For other purposes, the same condensation of statement would be a handicap, but not a serious one. With this qualification it is a simple and admirable recital of facts, particularly covering the developments in 1922, 1923 and 1924, the years to date during which conditions have been most free from the war. One may wish but scarcely hope that it will be carefully studied by all, both in and out of Congress, who will vigorously debate the McFadden Bill and any others that may be introduced. Instead the discussions will as usual be filled with emotion and be lacking in facts.

Necessarily the major part of the treatment is a recital of facts familiar to students of banking. A few are worthy of notice. The system is growing in strength. About 72 per cent of the banking resources is in the system. In 1924 there were 8,080 national bank members and 1,570 non-national bank members, a total of 9,650. Total resources were $4,812,143,000, of which $928,355,000 were earning assets. Gold reserves were $3,069,163,000, and Federal Reserve Notes in actual circulation amounted to $1,729,859,000.

Such figures are impressive, but interest concentrates more on some of the policies of the system and the changes that have occurred. From December, 1920, when the Federal Reserve Notes outstanding were $3,344,686,000, there

was a reduction to $1,862,062,000 on December 31, 1924, a remarkable decline when considered in connection with the large influx of gold that has occurred. There seems little reason to fear that the concentration of gold and of control in the hands of the banks will cause a dangerous inflation. In fact, the period is one in which the gold holdings of the banks have been increasing rather than diminishing.

Light on this movement and on the allied question of control is to be found in the chapter on Federal Reserve Notes. For two years past gold has been deliberately put into circulation in the form of gold certificates and Federal Reserve Notes have been withdrawn. In this way the dangers of inflation have been minimized so long as the gold remains in the country. As the demand for export purposes appears it will not be difficult to reverse the process and permit the exportation without a sharp contraction in the volume of circulation and a serious fall of prices.

This immediately raises the question of general control. Has this control been abused or is it likely to be abused in the future? Is the danger so serious as to warrant a modification of the law as a precaution against it? One may imagine possibilities, but it is hard to find anything to date that warrants concern. The Federal Reserve Board has been gradually feeling its way, and its policies in so far as formulated are clearly set forth by the author in various parts of his study. Two devices have been employed. One is the discount rate which can be altered as conditions warrant. The other is the use of open-market operations which is probably even more potent and more significant than Mr. Goldenweiser's treatment indicates. It is his view (p. 62) that "the sale of securities, when properly timed, may support the discount policy of the reserve banks, and the proper management and timing of open-market operations consequently constitutes a part of the technique of an effective credit policy." It is not yet fully demonstrated that the alteration of the discount rate is the leading influence in credit control, especially in the United States, and it may be that open-market operations are the more powerful device for that purpose.

Most important is the fact that the Federal Reserve System is assuming an ever increasing responsibility for the financial condition of the country. Other countries have for years been able to act as a unit, and it is fortunate that we are now in a similar position, particularly since we are coöperating with other countries, especially England, to our mutual advantage. The necessity for such coöperation is increasing rather than diminishing, and it is to be hoped that no changes will be made in our law that will check the development of this kind of international action.

ERNEST MINOR PATTERSON

Changes in the Size of American Families in One Generation, by Ray Erwin Baber, Ph.D., and Edward Alsworth Ross, Ph.D. Madison: University of Wisconsin Studies in the Social Sciences and History, Number 10. 1924. 99 pp.

The decreasing size of the native American family has recently attracted the attention of many students of social problems. The difficulty of the investigation, and the cost of collecting significant data, however, have prevented the subject from being studied in the adequate way which its importance warrants. For this reason the extensive study of Professor Baber and Professor Ross is a significant and welcome contribution. Previous studies have been at a disadvantage either because they have dealt with the families of college students and those o their ancestors alone, or, because, while drawing their material from a wider field, they have used groups in which the "present" generation was not wholly descended from the "past" with which it was compared. In this investigation the authors, while maintaining the family continuity, have been able so to extend the study as to make possible comparisons between various educational and occupational groups among the native American "middle-class" families of the Central and Middle West.

The book, after an introductory chapter, opens with an exposition of the methods used in obtaining the data. The third chapter, A Comparison of the Past and Present Generation, is the most important. In it seven topics are treated: (1) Childhood Environment, (2) Occupation, (3) Education, (4) Age at Marriage, (5) Birth Intervals, (6) Size of Family, and (7) Mortality of Children Under Five Years of Age. The headings of the remaining nine chapters are: (iv) Relation Between Education and Age at Marriage, (v) Relation Between Occupation and Age at Marriage, (vi) Relation Between Age at Marriage and Size of Family, (vii) Relation Between Education and Size of Family, (viii) Relation Between Size of Family and Mortality of Children Under Five Years, (ix) Relation Between Occupation and Size of Family, (x) Relation Between Occupation and Mortality of Children Under Five Years, (xi) Relation Between Education of Parents and Mortality of Children Under Five Years, and (xii) Comparison of the Size of Families in This Study with Dependent Families.

The data for this study were obtained through students, mainly of the University of Wisconsin. Questionnaires were given only to those students whose parents were living and had completed their families, and whose grandparents were native-born. The students sent the questionnaires to their parents, who filled out a separate one for each of themselves, and for each of the students' grandparents and married uncles and aunts. The grandparents of the students form the "past generation," and the parents, and married uncles and aunts whose families were completed, the "present generation" of this study. The average date of birth of the past generation was 1836, and of the present generation 1866.[1] Data were obtained from about 750 families of the past generation and 2,500 families of the present generation.

When the representative method is used, it is necessary to define the group to

[1] The arithmetic mean is the only average used in this study.

be studied and to select carefully a body of data which will be typical of it. The authors believe that their material represents a middle-class element, and that for three reasons: (1) It was drawn from native American stock, (2) dependents were excluded, and (3) it was drawn from families in which at least one person of the connection was a student in some university where the proportion of the wealthy is small.

It is not certain that the data presented may be taken as a representative sample of the middle-class in the Middle-Western States. On page 9 the authors say, "This is, in the main, a study of families of the Middle West," and again on page 10, "it is very clear that this study is of the families of the Central United States." From the materials presented, the reader cannot know accurately the territory for which the study does speak. Although in each case the questionnaires asked for the state in which childhood was spent, the authors have not published this information. The authors know the state of residence of the students' parents, who form about one-sixth of the present generation. Of this one-sixth (416 families) whose place of residence is known, 36 per cent live in Wisconsin, and 78 per cent in the Central and Middle-Western States. The distribution of some of the remainder is: California 7, Colorado 6, Connecticut 2, Idaho 2, Maine 4, Maryland 6, Massachusetts 15, New Jersey 5, New York 17, Oregon 2, Pennsylvania 6, Texas 2, and Washington 3 families. Thirty-six states are represented by one or more families (p. 10). It is reasonable to suppose that those families whose place of residence is not known are more widely scattered than those whose place of residence is known. It seems clear, therefore, that of the present generation something less than 75 per cent live in the Middle and Central West, and certainly a smaller proportion of the past generation lived in that territory.

It appears unfortunate that the authors should have prejudiced the value of their sample by obtaining one-quarter of their data from territory with which this study is not concerned. The authors may believe that there is little difference between the various parts of the North, but of this there is no indication.[1]

In the families of the past generation, the average number of children was found to be 5.44 (p. 24). This figure is subject to two limitations: (1) "all childless families of that generation were automatically excluded from this study," and (2) "It is obvious, moreover, that the smaller families of a generation ago would stand a smaller chance of getting into the study" (p. 24). The average number of children in all families of the present generation is 2.80 (p. 26), and that of fertile families is 3.35 (p. 25). "The figures representing the 'present generation' we can handle w th full assurance, as they are not subject to the same limitations."

The authors believe that by using the average number of children of fertile families of the present generation and the figure for the past generation, "we can reach a comparison that has the possibility of only a very slight error" (p. 24). Thus they find that between the past and present generation the average number of children per fertile family has decreased 38.42 per cent (p. 26). As the smaller families are probably inadequately represented in the past generation of this

[1] For implication to the contrary see pp. 60, 61, 62.

study, the reviewer is not convinced that the error is very slight. When 38.42 per cent is given as an estimate of the percentage decrease in the number of children in all families, the probability of increasing sterility (which this study cannot measure) makes the figure appear more reasonable. The biases would then tend to cancel each other.

On the assumption that the percentage decrease in the average number of children of all families is the same as that of fertile families (38.42), and that this rate of decrease continues, in two generations the average number of children per family will be 1.05.

"And while this picture may be too dark, it is certainly not lightened by the fact that 13 per cent of the families of the 'present generation' are infertile. . . . And when we note that 17.7 per cent more are one-child families, the trend toward extinction is evident" (p. 27).[1]

When we ask ourselves, what chance our middle-class native stock has "in competition with their prolific neighbors from abroad," it is well to remember that stocks with high birth-rates, on entering this country, diminish their size of family with great rapidity—since 1900 probably with greater rapidity than that of native families. The fact remains, of course, that families of the foreign born have been, and are, distinctly larger than those of the native stocks.

The authors have frequently based their conclusions upon what the reviewer believes to be inadequate samples. Though at times the authors point out this inadequacy, they continue to use the data as a basis for their conclusions. For example, on page 78, we find, concerning the decreasing number of children in the fertile families of the unskilled laborers: "Even making allowance or inaccuracy due to the smallness of the sample it is still evident that the decline in their birth-rate is great." This decline (44 per cent) was computed from figures of the average number of children of fertile families taken from samples of 12 families in the past and 37 families in the present generation. The average age at marriage of women college students of the past generation was found to be 22.08 years, from a sample of 40 cases. Women graduate students of the present generation are found to have an average age at marriage of 31.64 years. The

[1] A rough comparison between the percentage decrease in the number of children per family of this study, and that of the white population of the North Central States, may be obtained by use of the census data. The past generation of Baber and Ross' study was marrying about 1860, and the present generation about 1890 (p. 22). Between these two dates in the North Central States, the number of white children under 5 years of age to 1,000 white women 15–44 years of age decreased about 30 per cent. These figures are affected by changes in infant mortality, in the proportion of the marrying population, in the proportion of infertile marriages, and in the proportion of women 15–44 years of age. It appears probable, therefore, that the decrease in the number of children per family for the white population of the North Central States has been less rapid than that for the group studied by Professors Baber and Ross.

In connection with the picture the authors have drawn of "the trend toward extinction," it is worth noting that, while between 1860 and 1890 the proportion of white children decreased nearly 30 per cent, between 1890 and 1920 this decrease was about 15 per cent. (Between 1910 and 1920 less than 1 per cent.)

These figures include those for foreign-born whites. As nearly as the reviewer is able to estimate, the proportion of children decreased between 1890 and 1900 less rapidly among the foreign-born whites than among the native whites. From 1900 to 1920 the reverse appears to have been true. If the group which the authors have studied is following in any sense the general trend for the entire native white population of the North Central States, it would seem probable that the rate of decrease in the number of children per family will be substantially cut in the next generation.

figure is obtained from a sample of 22 cases. Commenting on this latter figure (p. 32), the authors say: "The graduate training of the latter [women of the present generation] puts them $5\frac{1}{2}$ years behind their college sisters in the march to the altar,—those who get there at all! . . . And the highest step [of the educational stairs] on which the graduate women stand, is represented by the figure 31.6." Nowhere in this chapter is there reference to the possible inadequacy of the sample from which this figure is drawn. The reader is further amazed to find that in all the tables, no matter how small the sample, the computations are carried out to the second decimal place—often without a hope of significance.

There are many signs of carelessness in the book. For example, in defining the educational groups used, they say: "*Elementary* includes all those who had any part or the whole of a common school education, *i. e.*, up to and including the eighth grade; *high school* includes those who had at least one year in high school. . . ." (p. 18). In Table 1-E it appears, however, that the groups were intended to be mutually exclusive, and that a high school student, although he had also gone through the elementary school, would be ranked as *high school* only.

The title to Figure 7–B (p. 83) reads: "Percentage Decrease, from Past to Present Generation, in the Number of Children Born per Family in the Different Occupational Groups." The figure contains the percentage decrease for "fertile families" in the various occupational groups. There are at least fifteen inaccurate figures in the book, some of which may be misprints, but others are obviously the result of mistakes in calculation. In no case do these inaccuracies influence the argument.

The style of many passages rests ill at ease between the thin, brown covers of the University of Wisconsin Studies in the Social Sciences and History.

FRANK W. NOTESTEIN

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Elements of Physical Biology, by Alfred J. Lotka. Baltimore: Williams and Wilkins Company. 1925. x, 460 pp.

This work will, of course, interest primarily physicists, biologists and chemists. It can be recommended to statisticians, however, because of its brilliant treatment of the concepts of progression and irreversibility. Among American workers in bio-statistics, only Pearl and Reed have given adequate treatment to problems of growth. In the contemporary literature of economic statistics, the growth-concept has been cheaply treated. A careful study by American statisticians of Dr. Lotka's book may lead to a reconsideration, and perhaps the abandonment, of prevailing approaches to the phenomena of growth and covariation in historical series. A reading of the first sixty-three pages will prepare the statistician for an understanding of the Verhulst-Pearl-Reed formula.

At the outset, Dr. Lotka insists upon rigorous definition. He points out that certain pseudo-problems of science often owe their origin to a false masquerade as problems of fact when they are in essence only problems of definition. It would

be a wholesome discipline for statisticians to catalogue existing discussions in our own and in other statistical journals from this point of view. Real progress in the statistical discussion of social problems rests essentially upon the establishment of judicious definitions. The statistician could supplement Dr. Lotka's observations by reflection upon the introductory pages of the "*Principia Mathematica*," by Whitehead and Russell.

Dr. Lotka's distinction between *admissible* and *expedient* definition is important (p. 3). He warns against the current practice of finding or making names and then gaily setting forth on an expedition to find the thing or things that should answer to these names. This happens to be a failing of the contemporary "wodget" school of literary statistics. Professor Irving Fisher's recent comment on the fundamental inadequacies of the statistical literature on so-called "business cycles" reminds one of the comment of H. G. Wells on the inherent bias of the human mind to make names and forthwith to imagine something answering to the name:

> . . . If I say *wodget* or *crump*, you find yourself passing over the fact that these are nothings . . . and trying to think what sort of a thing a *wodget* or a *crump* may be. You find yourself insensibly, by subtle associations of sounds and ideas, giving these blank terms attributes.[1]

Dr. Lotka's book deals largely with the problems of organic evolution. Evolution is defined as "the history of a system undergoing irreversible changes." The study of these changes, involving as it does the adequate treatment of the concept of progression and irreversibility, presents problems of method which are in large measure within the realm of probabilities or statistics. In Chapter III, the most difficult one in the book, Dr. Lotka presents the statistical meaning of irreversibility. Having rigorously established his definitions, Dr. Lotka then proceeds to outline the program of physical biology. This branch of science is defined as the application of physical principles in the study of life-bearing systems as a whole; it is thus a branch of the higher discipline of the general mechanics of evolution, the mechanics of systems undergoing irreversible changes in the distribution of matter among their several components. Where attention is concentrated upon aggregates of individuals as components of life-bearing, evolving systems, we are dealing with a bulk-mechanics. Bulk effects thus observed are of the nature of a *statistical* manifestation, and their study constitutes a statistical mechanics. Dr. Lotka then discusses successively the stoichiometry of evolution which deals with the transformation of matter viewed merely from the standpoint of changing masses of the components. Then he treats of the energetics or dynamics of evolution, discussing the statistical treatment of the dynamical problems presented by aggregates of living organisms. Under stoichiometry, the discussion is further divided into (a) the *kinetics* of evolution, that is to say, the systematic expression of the velocities of transformation in relation to the masses of the several components and, (b) the *statics* of evolving systems where velocities of transformations are zero, or vanishingly small. Under this head fall the treatment of equilibria, moving equilibria and displaced equilibria.

[1] H. G. Wells, *First and Last Things*, 1908. p. 32.

Dr. Lotka's derivation of the Verhulst-Pearl-Reed equation occurs in a discussion of the fundamental equation of kinetics, having a single dependent variable. He takes the general form of the fundamental equation:

$$\frac{dX_i}{dt} = F_i(X_1, X_2 \ldots X_n; P, Q) \tag{1}$$

which, with a single variable X, may be written:

$$\frac{dX}{dt} = F(X) \tag{2}$$

and applies it to a special case where one particular biological species or group of species grows actively while conditions otherwise remain substantially constant. This is essentially what has occurred in human populations. Applying equation (2) to this problem, Dr. Lotka considers first the equilibrium equation:

$$\frac{dX}{dt} = F(X) = 0 \tag{3}$$

By suitable processes, the author developed this equation, which has been used by Professors Pearl[1] and Reed in fitting a curve to the observed data of population growth of the United States, into:

$$X = -\frac{\frac{a}{b}}{1+e^{-at'}} \tag{4}$$

Writing in the constants, this equation became for Pearl's problem:

$$N = \frac{197{,}273{,}000}{1+e^{-0.03134t'}} \tag{5}$$

where N = the calculated number of the population at time t' (in years) reckoned from April 1, 1914, t' being negative for dates anterior to this.

This equation seems to be suited to investigations of growth phenomena where factors other than growth remain constant, as was evidenced by Pearl's fit of this formula to an experimental population of fruit flies and Thornton's use of it in studying the growth of a bacterial colony. Dr. Lotka continues his discussion of the fundamental equations of kinetics in succeeding chapters, using a wide range of examples from the field of bio-statistics.

Similarly, practical uses are made of the equations representing equilibrium, with examples drawn from the Ross malaria studies, the relations between birth- and death-rates, the population of animals slaughtered for food, from meteorology and geophysics, and the problem of nitrogen and phosphorus supply of soils.

One word of caution may not be out of place. It must be remembered that, in the words of Karl Pearson, "in dealing with any natural phenomenon the mathematician has to simplify the conditions until they reach the attenuated character which lies within the power of his analysis." To those who understand

[1] *Proceedings*, National Academy of Sciences, 1920. Vol. VI, p. 275.

the spirit of the mathematical method this admission is in no wise alarming. Others may at times be misled by mathematical symbols, the precise underlying assumptions of which they fail to understand. For this reason the caution which Dr. Lotka adds to his brief discussion of the Ross malaria equations (p. 83) should be well noted: "The Ross equations refer to a rather highly idealized case, a constant population both of men and of mosquitoes. The latter could be even distantly approached only in the tropics. There is room here for further analysis along more realistic lines."

In Chapter XXIV, Dr. Lotka reads into the fundamental equations of kinetics the dynamical or energetic implications, by calling to mind that in their physical relations the components, whose masses X appear in the equations, are energy transformers. This leads to further chapters on the relation of energy transformers to available sources of energy, to correlating apparatus in the transforming process, adjustors of the process, the rôle and function of consciousness, and to a general "Conclusion."

Undoubtedly the fourth section of the book, Dynamics, contains Dr. Lotka's most original and most valuable contribution. The point of departure is that certain of the components of the evolving system which we have before us in nature are *energy transformers*, namely the biological organisms. Thus the dynamics of this evolving system presents itself as the statistical dynamics of a collection of energy transformers possessing certain particular properties. Whereas in the classical theory of statistical dynamics, as applied for example to the colliding molecules of a gas, it is usually a fundamental assumption that there is no appreciable correlation between the movement of two molecules except at an actual collision; on the contrary, in the interplay of energy transformers of the type denoted by the term "*organism*," such correlation not only exists, but becomes the central feature in the course of events. This is the spirit in which the section on Dynamics and Chapter XXVI on The Correlating Apparatus are conceived and should be read. In justice to the author, the heading "*Statistical Mechanics of Systems of Organisms*," which now appears on page 358, should have been placed at the beginning of Chapter XXV, for the matter that follows is a first step—proverbially the hardest—toward the development of the mathematical method applicable to a difficult problem.

A careful reading of the concluding Chapter XXXIV is recommended because of its bearing upon the philosophy of economics.

The plan of the work was laid about 1902 and its development was outlined in a series of publications, the first of which appeared in 1907 in the *American Journal of Science*. The last stage of the work, which consisted of arranging the matter in collected form, was carried out at the Johns Hopkins University upon the invitation of the Department of Biometry and Vital Statistics. The book represents more than twenty years of work in the collection of factual material and in the systemization of method.

The book is dedicated to the memory of John Henry Poynting, the British physicist, noted particularly for his contributions to our knowledge of the line of propagation of energy—the Poynting vector—in the electromagnetic field. But Poynting, himself a pupil of the great Clerk Maxwell, was a man of broad in-

terests, and among his collected papers will be found several statistical studies. Something of this breadth is discernible on every page of the book under review.

It is most fortunate to have a physicist enter the field of quantitative biology and infuse into this branch of knowledge something of the spirit that has come down to him by oral tradition from the great lights of the last century. There is much in the book that should prove inspiring, especially to the younger men entering the statistical field, whose minds are as yet plastic and receptive to a novel point of view.

E. W. KOPF

Housing: The Facts and the Future, by Harry Barnes. London: Ernest Benn, Limited. 1923. 450 pp.

In spite of its length and its concentration on the quantitative details of a social problem which in many respects is entirely different from the corresponding one we face in this country, this book is intensely interesting even to an American reader. Its author, Major Harry Barnes, is one of the newer school of Liberals, who has discarded the traditional *laissez-faire* for the doctrine of collective responsibility and who believes that the housing acts of today are in direct apostolic succession to the great Poor Law legislation of Elizabeth and the Education Act of 1870. His real interest, and it is a burning one, is in the human aspects of this problem—so much so that he half apologizes for devoting a book to counting costs and reckoning rents instead of to the things which may neither be reckoned nor counted. This social zeal touches with eloquence at times a style which is always facile and engaging, and turns, for instance, what in other hands would be an arid discussion of the "nature and limitations of statistical units of measurement" into an interesting chapter on "Dwellings and Rooms."

But the book is not the product of mere sentiment. With the earnestness of the social reformer has been coupled balanced judgment, rich experience, painstaking industry and technical competence. The author's competence and experience are sufficiently attested by the title page. Among other things, Major Barnes is Vice-President of the Royal Institute of British Architects, Fellow of the Surveyors' Institute, Honours Medallist in Building Construction, and during his parliamentary tenure was Chairman of the Housing Committee of the Independent Liberal Party in the House of Commons and member of a number of select committees dealing with land values, rent restriction, etc.

In his view, "the housing problem" is "to provide, after allowing a proper margin for the movement of the population, a sufficient number of separate dwellings of a desired standard, with adequate accommodation and the proper conveniences, at a rent within the range, and in positions conveniently near to the work of the wage-earners of this country." His own problem in writing the book was to collect, classify and interpret the statistical data which would enable answers to be given to the questions whether such provision for housing could be made and whether it had ever been made in the past. These data have been compiled and presented, probably more completely and more conveniently than anywhere else, in the statistical tables of the last third of the book. The rest of

the volume contains the author's own interpretation, developing the main thesis that a century of unregulated individual effort has hopelessly failed to meet the housing problem, that it is likely to perform even less adequately in the future, and that consequently without the most vigorous, continuous, state-organized, and state-aided effort the housing position at the end of the present decade will be worse than it was in 1921.

With questions of British political policy we are not here primarily concerned. The author has made out a fair case for most parts of his constructive program, but the British seem at last to have reached their decision on housing policy. In the Housing Act of 1924 supplementing that of 1923, they have now a well-coördinated and well-planned body of housing law which the authorities may regard as reasonably fixed and settled instead of merely experimental and idealistic. It does not go as far as Major Barnes would go, but at least it recognizes the necessity of organized state effort and substantial subsidy.

The author's statistical method and the data which he has collected are of more interest to readers of this JOURNAL, though the reviewer does not consider justified a detailed critical appraisal. His general method is to go to the Census Returns, the Returns of the Registrar-General of Births, Deaths and Marriages, and the Returns of the Inland Revenue Commissioners relating to the number of houses liable to and exempt from inhabited house duty; secure the facts in regard to population, marriages, families and houses for the period 1801 to 1911; and then establish "an average rate of increase and variation by which the past may be judged and the future estimated." No attempt is made to determine long-run trend by fitting a straight line to the data according to the method of least squares or in any other way. By applying the factor of judgment to a simple inspection of decennial averages and percentages of variation, Major Barnes reaches the conclusion that during the decade 1911–1921, war arrears of 500,000 houses in Great Britain were added to pre-war arrears of 800,000 "surplus families" (*i. e.*, families for which no structurally separate dwellings were provided) and that before 1930 a million new houses must be provided if pre-war and war arrears are not to increase. Then follows a detailed examination of the resources of the building industry in men and materials and also a careful analysis of the various items entering into the cost of a pre-war and a post-war house. Increases in cost of materials and in building wages are shown in most instances as great as or greater than corresponding increases in the United States. Interesting comparisons can be drawn between the declines in the various skilled crafts in the two countries. By a strange coincidence, the figure which Major Barnes gives as the average percentage to total population of men engaged in the building industry is exactly the same as the corresponding figure for the United States in 1910—2.56 per cent.

The author usually keeps well in mind the limitations of his data and, as already indicated, has an especially interesting discussion of the difficulties involved in his units of statistical measurement. The term "house," "though picturesque, homely and familiar, is inexact, ambiguous and unscientific, and the proper term to employ in a serious Housing inquiry is not 'house.' For what is a house? Has it one room or ten rooms? Is it the single-storied cottage of Sunder-

land, the double-storied pair of flats in Newcastle, or the six-storied, twelve-flatted tenements of Edinburgh?" What indeed is a room? "Are sculleries and bathrooms to be counted rooms? Are box rooms going to be counted in? Must there be a certain number of cubic feet in contents before it is to be called a room?" How many rooms is each person to have before he is to be counted as being housed? Are the rooms grouped properly to fit the persons grouped as families? If the rooms group all right, do they sub-group equally well to fit the sub-groups produced by the sexes? Are the rooms and the dwellings where the people are? Do the rents fit the earnings of the groups? Such difficulties of definition and measurement receive due consideration.

Criticism is probably more justified when directed against some of the assumptions of his general method. The use of an average based on ten or eleven decennial periods would frequently not be a safe basis for forecasting in this country, where changing normals over shorter periods have been the rule. It is probably more justified in the case of a country like England where growth has been more regular and is likely to continue so. The form in which the tables are presented is open to criticism. It is irritating in studying a series of columns to have to refer by number to a list of five or six headings given at the top of the page. The notes on the various tables would also be more convenient if found at the beginning or end of each table instead of being collected together in groups every ten or fifteen pages.

A series of appendices containing among other things a synopsis of the various estimates of housing needs made during and since the war add to the value of the book. The form of questionnaire used for the survey of housing needs under the Town Planning Act of 1919 is given in full. It illustrates most of the errors that could be made in framing such a questionnaire. For instance, it is much too lengthy, it asks for information which could not be secured except by the full machinery of a census, it uses ambiguous terms, and its adequacy depends largely on the political and other views of the persons filling in the returns. In addition to these defects less than two and a half months were allowed for the survey. The results were considered so inaccurate that a full report of the survey was never printed.

W. C. Clark

The Rôle of the State in the Provision of Railways, by H. M. Jagtiani, M.Sc., with an introduction by Sir William Acworth, K.C.S.I. London: P. S. King & Son, Ltd. 1924. xi, 146 pp.

This little book was originally prepared as a thesis for the degree of M.Sc. in the University of London, and offers one additional bit of evidence that Indian contributions to economics are deserving of serious attention. The method used is that of an historical survey, and the countries selected are England, Prussia and India. The policies followed fall into three well-marked divisions: the *laissez-faire* policy, government guarantee of the earnings of private companies, and government construction. The advantages of these policies are judged from the

standpoints of furnishing adequate facilities, distributed in proportion to the need, with a minimum of wasteful and unnecessary building, and organized into a well-coördinated whole. Indeed, the importance which the author attaches to this factor of coördination, and the manner in which he expresses it, are such as to give the impression of prejudging the case against the *laissez-faire* policy at the start.

The chief criticism he makes on the English experience is the waste resulting from uncoördinated development and from speculative overbuilding which, he holds, far from having been a blessing in promoting more rapid growth of facilities, came perilously near retarding growth by weakening railroad credit. The evidence cited is perhaps too largely the opinions of experts as expressed in governmental hearings and lacks citation of actual cases. But of course this sort of evidence is peculiarly difficult to secure and appraise, and particularly so for a foreigner.

In his description of the Prussian experience the author follows the figures of mileage constructed under different policies, and pays particular attention to the relative mileage in the less highly developed parts of the country as compared to those with denser population and more industrial development. "The inevitable result of uncontrolled competition in construction is that whereas the private enterprise is abundant in parts of the country which are rich and already well served, the poorer districts are left to the government to be developed." In particular. East Prussia, even as late as 1883–1884, had not only fewer kilometers of line per 100 square kilometers, but also per 10,000 inhabitants. The state system is shown to have remedied this, and while it did not give the sparsely settled areas as much railroad per 100 square kilometers as the other regions (which would hardly be reasonable) it did give them consistently more road per 10,000 inhabitants.

An interesting incidental point is the discussion of the necessity of placing the light railways under a different administration from the standard lines, lest standards of construction appropriate to the heavier lines should find their way into the construction of the lighter ones, where they would be quite out of place and could only waste funds.

In India we find the state compelled to shoulder the burden. We also find a new element entering into the situation, namely, the relations between the Indian government and the home government in England. To one not familiar with the constitutional apportionment of powers between various officials and ruling bodies, as well as with the personnel, morale and capabilities of the different organizations, it is decidedly difficult to judge of the merits of the contest—for such in effect it appears to have been. The author is consistently favorable to greater freedom of action for the government of India. In general, he leans toward the system of government construction as against either the system of guarantees or that of unrestricted private construction.

While the style of the book betrays here and there the fact that the author is writing in a language not native to him, these indications are surprisingly few and slight. At times, however, they introduce minor obscurities of meaning. But he has selected his material so as to give a condensed and interpretative picture, and

a large part of the story will be new to American readers, including the Indian history and much of the Prussian material.

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Russian Currency and Banking, 1919–1924, by S. S. Katzenellenbaum. London: P. S. King & Son, Ltd. 1924. x, 198 pp.

This brief summary of Russian currency and banking developments by Professor Katzenellenbaum of the University of Moscow devotes most of its 198 pages to a discussion of developments since the autumn of 1917, when the Bolsheviki came into power. The statement of pre-war conditions in Chapter I is brief, and the discussion in Chapter II is devoted to an exposition of the writer's views on monetary theory. Russia's currency and banking experiences in recent years will afford great opportunities for research for the student of monetary theory and practice. At the outbreak of the war Russia had the largest gold reserve in existence and a currency that had remained at par for a quarter of a century; in 1924 the reserve had been almost entirely dissipated and the various issues of paper money, after having repeatedly gone through a process of dropping ciphers, had finally ceased to serve as a medium of exchange, and a gold secured currency was once more introduced. Of more significance to the student, however, than the financial developments themselves will be the influence of these developments on the welfare of the people, and on the functioning of the economic mechanism. It seems to the reviewer that such a student will find but little help in a study of the present volume, which is incomplete and over-simplified in its statement of facts, and both academic and dogmatic in its theoretical approach.

E. A. G.

Public Employment Offices: Their Purpose, Structure and Methods, by Shelby M. Harrison, in collaboration with Bradley Buell, Mary La Dame, Leslie E. Woodcock, and Frederick A. King. New York: Russell Sage Foundation. 1924. xvii, 685 pp.

When this tome of 700 pages came from the press, the publicity which it received centered largely about the unemployment estimates contained within it. But the impression that the book is a statistical study is erroneous; it is 95 per cent non-statistical. The chief aim of Mr. Harrison and his associates was simply to give a critical exposition of existing placement methods and agencies, the policies and practices followed, and recommendations for improving their efficacy through exercise of common sense. Even this is "a large order," owing to the chaotic variations of practice now in use among employment offices; but the authors have nevertheless achieved very noteworthy success in filling the order.

Although the authors emphasize the *public* employment agency, they do not overlook other forms; nor do they concentrate their energies on a plea for eliminating private agencies and inaugurating at once complete public organization under centralized control. They do plead for a closer approximation to an organic national structure than exists today, but they believe in trying a jointly managed

federal-state-local service similar to that set up during the war. Moreover, even that plea is not central, but rather incidental to the discussion of public agencies as they are, from the private and the public municipal type on up.

The introduction, by Mr. Harrison, and Part One, by Mr. Buell, simply set up the background for the more central matter in Parts Two, Three, and Four. Briefly stated, the purpose here is to show the two fairly obvious facts, that a disorganized labor market is undesirable, and that its organization must in the nature of the case be worked out under impartial public auspices, preferably federal in scope.

The proper organization and functioning of the *local* office is treated in Part Three, written by Mr. Woodcock and Miss La Dame jointly. This embodies an able, sensible application of recognized business principles, with which the officers and staffs of many local offices are sadly unfamiliar. Functionalization, process-analysis, preparation and use of suitable operating records, methods of interview, placement and clearance, principles for selection and control of office personnel form the meat of the matter in Part Three. It is possibly the best, and probably the most immediately useful, section of the book.

Mr. King's Part Four treats of special-service groups, such as juveniles, negroes, professional workers, immigrants, handicapped workers, and casuals. This treatment is sympathetic, thorough, and at points even acute. It is based upon a comprehensive analysis of past experience encountered among such groups by *municipal, state and other bureaus*. The lessons drawn from that experience should be helpful to employment-office managers or lawmakers, whatever the territorial scope of their authority.

In Part Two, which to our mind would have more logically formed the last Part and the climax of the book, a *national* public employment office system is planned as to organization and operation. Miss La Dame draws freely upon Canadian and British experience, as well as that available for this country in the work of the Bureau of Immigration and the United States Employment Service. Of necessity she resorts to considerable speculation also, in piecing together the evidence as to the probable workability, in the long run, of various forms of joint control. Problems of political exigency are studied side by side with those of operating efficiency. In sense of detachment and in speculative shrewdness, these nine chapters—especially 6 and 7—rank high; at points they almost remind one of Bryce or Madison.

The book has minor faults. The bulkiness resulting from so scholarly a treatment, together with a certain amount of repetition in the matter written by the several collaborators, may limit its usefulness to some readers who should familiarize themselves with such a work. Inadequate critical selection in preparing the long bibliography, and loss of freshness at points, owing to the length of time (five years) elapsing between the commencement and the conclusion of the researches, are also noticeable. But we have no hesitation in regarding it as a high-grade synthesis of available writings on the subject, supplemented by new field work ably described and criticised. Without question it is among the best of the Russell Sage publications.

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